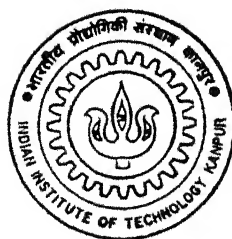


REGION IDENTIFICATION AND CLASSIFICATION OF MULTITEXTURED IMAGES USING WAVELET PACKETS

by

Major B V B Prasad



DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

March, 1996

EE
1996
M
PRA
REG

REGION IDENTIFICATION AND
CLASSIFICATION OF MULTITEXTURED
IMAGES USING WAVELET PACKETS

A Thesis Submitted

in Partial Fulfilment of the Requirements

for the Degree of

MASTER OF TECHNOLOGY

by

MAJOR B V B PRASAD

to the

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

March 1996

21 MAR 1996
CENTRAL LIBRARY
I. I. T., KANPUR
Doc No. A. 121205

EE-1996-M-PRA-REG



A121205

Certificate

It is certified that the work contained in the thesis entitled REGION IDENTIFICATION AND CLASSIFICATION OF MULTI TEXTURED IMAGES USING WAVELET PACKETS, by Major B V B Prasad, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

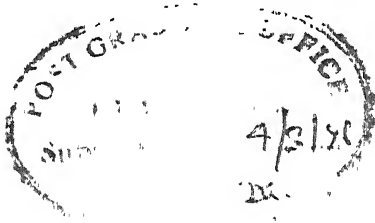


Dr. Sumana Gupta

Associate Professor

Department of Electrical Engineering

I.I.T. Kanpur



Abstract

Textures provide important characteristics for a large number of applications in image processing. The two important characteristics of texture are *Coarseness* and *Directionality*. Though various statistical and structural techniques have been in use for a long time, the multiresolution approach is now being increasingly applied for texture analysis. In multiresolution analysis the wavelet theory is an effective tool as it can effectively emulate the human visual system .

In this thesis wavelet packet transform has been applied for texture classification. Two approaches have been studied. The Progressive classification approach and the Complete Energy Map. It was found that the complete energy map approach gave a 100 % classification at level L-1 and L level respectively for an L level decomposition of the given image.

An algorithm for region identification and classification, for multitextured images has been proposed. An attempt has been made to combine the region identification and classification of the various textures in an image, by exploiting the wavelet packet energy signatures of the textures. As the textures have distinct signatures, the dominant subspaces of a texture present in the image are utilized to identify the region of the chosen texture. To confirm the correctness of the region, we extract the region and classify it. The advantage with this algorithm is that it is computationally simple. In region identification though the reconstructed region is of inferior resolution, this algorithm effectively identifies the region of the chosen texture if it is present.

Acknowledgement

I would take this opportunity to express my sincere gratitude towards **Dr. Sumana Gupta**, my thesis supervisor for her invaluable guidance. She has always been very patient and encouraging regarding all my queries and doubts during my thesis work. I would also like to thank Major Sharad Shukla, Sanjay G. Joshi, Gomathi Sankar, M. Mahaboob Basha , V. R. Babu who were a great support during my thesis.

Finally, I would like to express my gratitude towards the Indian Army and Corps of Signals in particular for having given me the opportunity to study in this premier institute of the country.

Major B V B Prasad

Dedicated

to
My Family

Contents

1	Introduction	1
1.1	Importance of Texture Analysis	1
1.2	Review of Texture Analysis	1
1.2.1	Statistical Approach	2
1.2.2	Structural Methods	3
1.2.3	Multiresolution Analysis	4
1.3	Organisation of Thesis	4
2	Wavelet Transform And Theory	5
2.1	Introduction	5
2.2	Wavelets	5
2.3	Construction of Wavelets	6
2.4	Wavelet Decomposition	7
2.5	Wavelet Packet Decomposition	12
3	Texture Classification And Region Identification	17
3.1	Concept of Wavelet Packet Signature	17
3.2	Selection of Filters	19
3.3	Spatial Characterization of the	19
3.4	Minimum Distance Classification	20
3.5	Classification Algorithm	21
3.5.1	Progressive Classification	21
3.5.2	Classification Using Complete Energy Map	22
3.6	Implementation for Single Textured Images	25

3.6.1	Results	28
3.7	Region identification and Classification	30
3.8	Image Rescaling	31
3.9	Results and Discussion	31
3.10	Highlights of The Proposed Method	33
4	Conclusions	48
4.1	Suggestions For Future Work	49

List of Figures

2.1	Wavelet decomposition	8
2.2	Reconstruction from wavelet coefficients	8
2.3	2D Wavelet Decomposition	10
2.4	2D Wavelet Reconstruction	11
2.5	1D Wavelet Packet Decomposition	13
2.6	2D Wavelet packet decomposition	15
2.7	Wavelet basis	16
2.8	Wavelet packet basis	16
3.1	Frequency plane division of the wavelet transform	18
3.2	Flow Chart for Training Phase of Progressive Classification	23
3.3	Flow Chart for Classification Phase of Progressive Classification	24
3.4	Flow Chart for Complete Energy Map Classification	26
3.5	Flow Chart for Classification Phase of Complete Energy Map Classification	27
3.6	Flow diagram region identification	32
3.7	Display of wavelet coefficients at level1 and level2	35
3.8	Daubechies 4-tap Wavelet function	36
3.9	Daubechies 4-tap Scaling function	36
3.10	Daubechies 20-tap Wavelet function	37
3.11	Daubechies 20-tap Scaling function	37
3.12	Energy distribution Image1	38
3.13	Energy distribution Image2	38
3.14	Energy distribution Image3	39
3.15	Energy distribution Image4	39
3.16	Energy distribution Image5	40

3.17 Energy distribution Image6	40
3.18 Energy distribution Image7	41
3.19 Energy distribution Image8	41
3.20 Energy distribution Image9	42
3.21 Energy distribution Image10	42
3.22 Classification rate Daubechies 4-tap Filter	43
3.23 Classification rate Daubechies 20-tap Filter	43
3.24 Region Identification Two Textured Images Set-1	44
3.25 Region Identification Two Textured Images Set-2	45
3.26 Region Identification Two Textured Images Set-3	46
3.27 Region Identification Three Textured Images	47

List of Tables

3.1	Variance of Scaling functions	19
3.2	Daubechies 4-tap filter wavelet coefficients	28
3.3	Daubechies 20-tap filter wavelet coefficients	29

Chapter 1

Introduction

1.1 Importance of Texture Analysis

Textures are considered to provide important characteristics for surface and object identification from aerial photographs, biomedical images and many other type of images. Their analysis is fundamental to many applications such as industrial monitoring, remote sensing etc. Though a lot of work has been done in the field of texture analysis encompassing the area of segmentation, classification, feature extraction etc, it is still considered an interesting but difficult problem in image processing.

1.2 Review of Texture Analysis

Texture is defined as a structure composed of a large number of, more or less, ordered similar elements or patterns without any of them drawing special attention and thus offering a global unitary impression to the observer. The two major characteristics of a texture are *COARSENESS* and *DIRECTIONALITY*. The two major texture analysis approaches are *Statistical* and *Structural* [6]. Texture analysis essentially comprises of *TextureClassification*, *Segmentation* and *Texture Synthesis* .

1.2.1 Statistical Approach

On the statistical level Texture is defined by a set of statistics extracted from a large ensemble of local picture properties. These properties define the set of primitives for texture analysis.

- Cooccurrence Matrices [8] : The spatial grey level dependence method (*SGLDM*) is based on the estimation of the second order joint conditional probability density functions $f(i, j|d, \theta)$. Each function is the probability of going from grey level i to grey level j given the intersample spacing as d and the direction by angle θ . The estimated values are written in the form of a cooccurrence matrix. From the cooccurrence matrix several textural features like energy, entropy, correlation, local homogeneity etc are extracted.
- Gray Level Difference Method [9] : Let $g(n, m)$ be the image intensity for any displacement $d = (N, M)$, where N, M are integers. Let $gd(n, m) = |g(n, m) - g(n + N, m + M)|$. From the probability distribution function $f'(i, d) = P(gd(n, m) = i)$, with i being the intensity value, several texture features can be derived .
- Gray Level Run Lengths [10] : This method is based on computing the number of gray level runs of various lengths. A gray level run is a set of linearly adjacent picture points having the same gray level value. The element $r'(i, j|\theta)$ of the gray level run length matrix specifies the number of times a picture contains a run of length j for gray level i in the direction given by angle θ . Several features are extracted from the matrices calculated for different directions.
- Fourier Power Spectrum Analysis [11] : The image is analyzed using the Discrete Fourier Transform, which treats the input as periodic. The borders strongly influence the spectra as they are treated as abrupt edges. Related with the Power Spectral Analysis is the Autocorrelation function. If the primitives are small then the ACF will fall sharply. If the primitives are large then the autocorrelation function will drop off slowly. Thus the texture coarseness is reflected in the autocorrelation function.
- Laws Approach [12] : Laws approach to texture characterization consists of two steps. First microstatistic features are computed using 3×3 or 5×5 convolution

masks. Second, macrostatistic features are obtained over large windows. Then texture energy measures are defined from these convolutions and textures are thereby characterized.

1.2.2 Structural Methods

Textures with more regular structure lend themselves to structural analysis methods. On the structural level, a texture is considered to be defined by elements which occur repeatedly according to some placement rules.

- Methods based on primitive extraction only [13] : Various methods of extracting primitives based on gray level thresholding are used. The next step is then to measure several primitive features and use them to discriminate between the various textures. The texture classification is done using the first order statistics of the primitives.
- Syntactic Approach [14] : This considers that the natural texture results from an ideal texture according to some transformation rules. The picture is divided into fixed size windows and with some fixed tree structure. A tree grammar is then used to characterise the gray tones of each windowed pattern.
- Edge based approaches [11] : The coarseness of a texture can be represented in terms of the density of the edge pixels. Also edge detection is of fundamental importance in texture analysis because edges characterize boundaries and therefore are useful for segmentation, object identification etc. The various edge detection techniques are use of gradient operators, compass operators, laplacian operators, detection of zero crossing, stochastic gradients etc.
- Region based approaches [11] : The various techniques used are the clustering, and the split and merge approaches respectively . In clustering technique, seeds of different textures are identified and the region of homogeneous texture is built around these seeds. In split and merge, technique the image is split into equal sized blocks using the quad tree approach. Based on some property of the probability distribution function of the image, further splitting of the image is performed. Subsequently similar blocks are merged to get homogeneous textured regions

Model based approach [15] [16]: These methods are based on models which were originally developed for texture synthesis. They assume some kind of dependence that a pixel has on its neighborhood. These models can be based on the linear dependence as in case of autoregressive models or the joint probability density functions with Markov models.

1.2.3 Multiresolution Analysis

Though early work on texture analysis was based on statistical and structural techniques, a large amount of work is now being done by using methods based on multiresolution and multichannel analysis[17] [18] [19]. This is because the traditional techniques were limited by the inability to characterize textures at various scales. The study of human vision system suggests that the spatial/frequency representation, which preserves both global and local information, is adequate for quasi-periodic signals. Since a large no of textures can be viewed as quasi-periodic patterns they can be represented by concentrations of spatial frequencies and orientations. The wavelet theory has been under intensive study in the last few years as a tool for scale space analysis. Coifman, Meyer and Wickerhauser [20] have generalized the wavelet basis to include a library of orthonormal bases called Wavelet Packets, which permit a very flexible tool for texture analysis.

1.3 Organisation of Thesis

In this thesis we begin with a brief review of the wavelet and wavelet packet theory given in Chapter II followed by simulation and results of the application of wavelet packet theory to texture analysis in Chapter III. Finally in Chapter IV conclusion and scope for future work is discussed.

Chapter 2

Wavelet Transform And Theory

2.1 Introduction

An important problem of image processing is to have a representation that is well adapted to extracting the desired information from the image. The extraction of the information is done by the decomposition of the image using various transforms. The earliest method was the Fourier representation using the complex sinusoids. This was accomplished, by the orthogonal properties of the family of the basis functions, obtained from the sine and cosine functions. This was not localized in the spatial domain. To overcome this problem the Short Time Fourier Transform was used. The wavelet transform like the STFT provides localization in both the domains, but also has a further advantage that the window size changes with the frequency content of the image.

2.2 Wavelets

Wavelet transform of a signal implies the decomposition of a signal with a family of real orthonormal bases $\psi_{m,n}(x)$ obtained through translation and dilation of a kernel function $\psi(x)$ known as the **mother wavelet** i.e.

$$\psi_{m,n}(x) = 2^{-m/2} \psi(2^{-m}x - n) \quad (2.1)$$

where $m, n \in \mathbb{Z}$. Due to the orthonormal property, the wavelet coefficients of a signal $f(x)$ can be easily computed by the analysis formula

$$c_{m,n} = \int_{-\infty}^{\infty} f(x) \psi_{m,n}(x) dx. \quad (2.2)$$

To recover the signal from the wavelet coefficients, the synthesis formula is

$$f(x) = \sum_{m,n} c_{m,n} \psi_{m,n}(x) \quad (2.3)$$

2.3 Construction of Wavelets

To construct the mother wavelet $\psi(x)$, the scaling function $\phi(x)$ and multiresolution analysis is used. In multiresolution analysis, the space of square integrable functions $L^2(R)$ is decomposed into closed subspaces V_j with the following properties

- $V_J \subset V_{J+1}$: The coarser subspace is contained in a finer subspace.
- $\bigcap_{m \in \mathbb{Z}} V_m = 0$: Separation Condition.
- $\bigcup_{m \in \mathbb{Z}} V_m = L^2(R)$: Condition for Completeness.
- $f(x) \in V_m \leftrightarrow f(2x) \in V_{m-1}$: Scaling Property.
- There exists a scaling function $\phi(x) \in V_0$ such that

$$\phi_{m,n}(x) = 2^{-m/2} \phi(2^{-m}x - n), m, n \in \mathbb{Z} \quad (2.4)$$

- Since $\phi(x) \in V_0 \subset V_1$ and $\phi(2x)$ are a basis for the subspace V_1 , the scaling function can be written as a linear combination of $\phi(2x)$ and it satisfies the two scale difference equation [1]

$$\phi(x) = \sqrt{2} \sum_k h(k) \phi(2x - k). \quad (2.5)$$

The wavelet kernel $\psi(x)$ is given from the the scaling function by the equation

$$\psi(x) = \sqrt{2} \sum_k g(k) \phi(2x - k) \quad (2.6)$$

where

$$g(k) = (-1)^k h(1 - k). \quad (2.7)$$

2.4 Wavelet Decomposition

To perform the wavelet decomposition, a recursive algorithm is implemented. The explicit forms of $\phi(x)$ and $\psi(x)$ are not required as they depend on $h(k)$ and $g(k)$ respectively, given by equations 2.5 and 2.6. A J level decomposition can be written as

$$f_0(x) = \sum_k c_{0,k} \phi_{0,k}(x) \quad (2.8)$$

$= \sum_k (c_{J+1,k} \phi_{J+1,k}(x) + \sum_{j=0}^J d_{j+1,k} \psi_{j+1,k}(x))$ where coefficients $c_{0,k}$ are given and coefficients $c_{j+1,n}$ and $d_{j+1,n}$ at scale $j+1$ are related to the coefficients $c_{j,k}$ at scale j by the equations

$$c_{j+1,n} = \sum_k c_{j,k} h(k - 2n) \quad (2.9)$$

and

$$d_{j+1,n} = \sum_k c_{j,k} g(k - 2n) \quad (2.10)$$

where $0 \leq J$. Thus equations 2.9 and 2.10 provide a recursive algorithm for wavelet decomposition through $h(k)$ and $g(k)$ and the final outputs include a set of J level wavelet coefficients $d_{j,n}$, $1 \leq j \leq J$ and the coefficient $c_{J,n}$ for the low resolution component $\phi_{J,k}(x)$. Similarly the recursive formula for the synthesis of the function is based on the wavelet coefficients $d_{j,n}$, $1 \leq j \leq J$ and $c_{J,n}$ such that

$$c_{j,k} = \sum_n c_{j+1,n} h(k - 2n) + \sum_n d_{j+1,n} g(k - 2n) \quad (2.11)$$

Figure 2.1 illustrates the decomposition given in equation 2.9 and 2.10. The pair of filters H and G with impulse response $h(n) = h(-n)$, $g(n) = g(-n)$ are the quadrature mirror filters, corresponding to the half band low pass and high pass filters. Figure 2.2 illustrates the reconstruction procedure implemented by upsampling the subsignals c_{j+1} and d_{j+1} (inserting a zero between neighboring samples) and filtering with $h(n)$ and $g(n)$ respectively, and adding the two signals. The wavelet transform decomposition is performed recursively at the output of the low pass filter 'H'. Thus the wavelet transform decomposes a signal into a set of frequency channels that have narrower bandwidths in the lower frequency region.

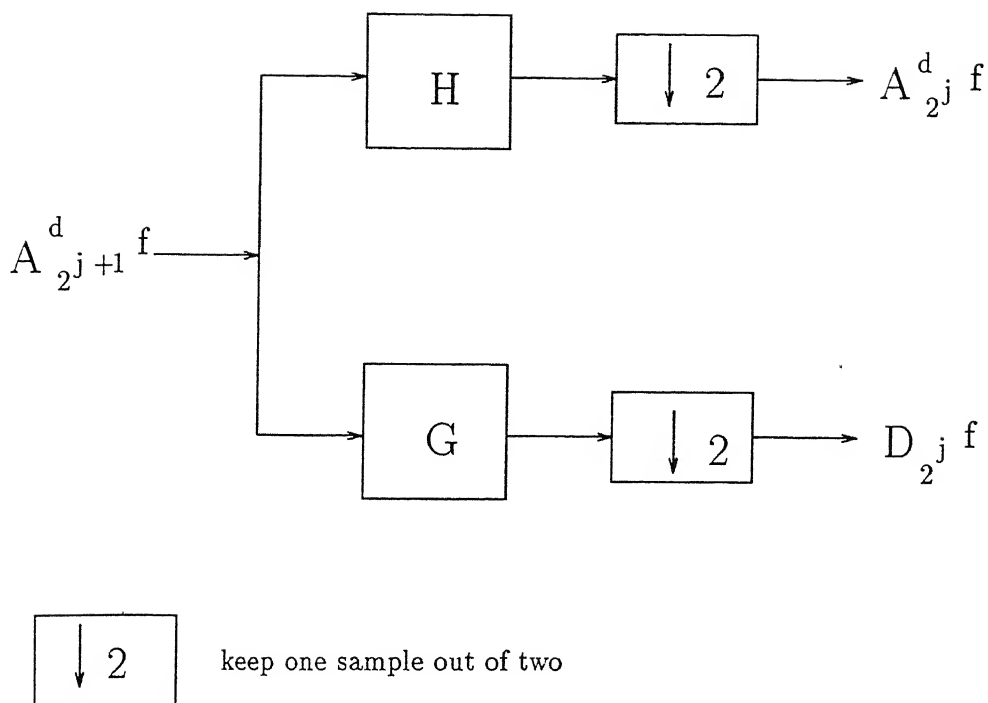


Figure 2.1: Wavelet decomposition

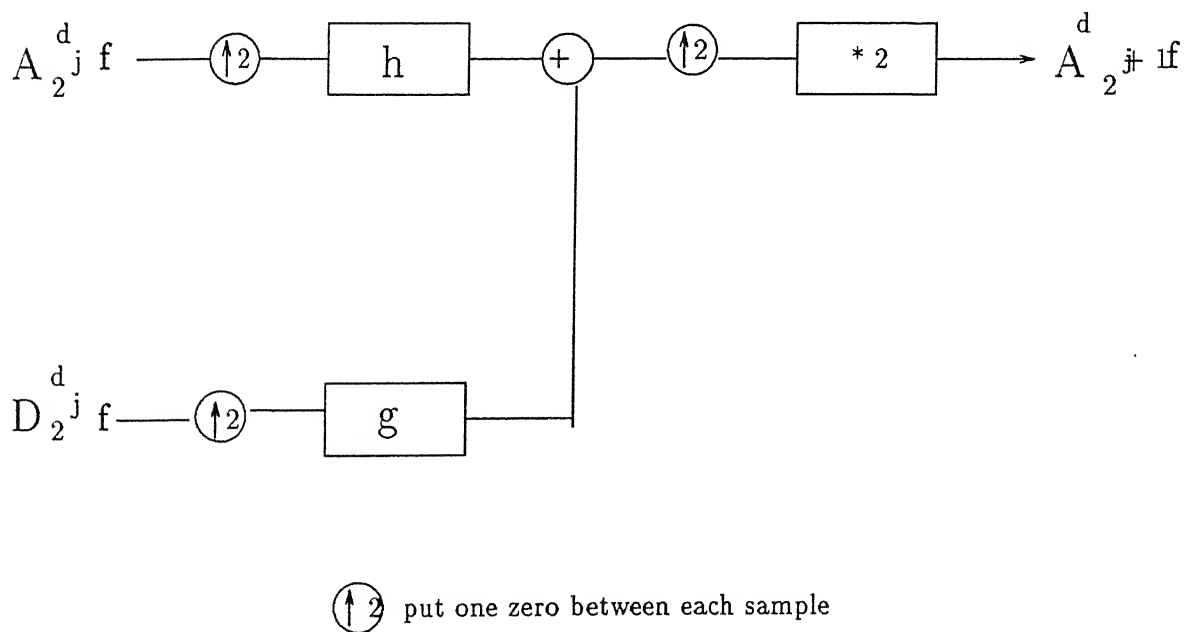


Figure 2.2: Reconstruction from wavelet coefficients

Two Dimensional Wavelet Decomposition. The wavelet decomposition can be easily generalized to any dimension $n > 0$ [2] . Extending it to two dimension for image processing, the signal is now a finite energy function $f(x, y) \in L^2(R^2)$. Let $(V_{2^j})_{j \in \mathbb{Z}}$ be the multiresolution approximation of $L^2(R^2)$. The approximation of the signal $f(x, y)$ at a resolution 2^j is equal to its projection on the vector space V_{2^j} . Let V_{2^j} be a separable multiresolution approximation of $L^2(R^2)$. Let $\phi(x, y) = \phi(x)\phi(y)$ be the associated separable two dimensional scaling function. Let $\psi(x)$ be the one-dimensional wavelet associated with the scaling function $\phi(x)$. Then the three wavelets are defined as

$$\begin{aligned}\psi^1(x, y) &= \phi(x)\psi(y) \\ \psi^2(x, y) &= \psi(x)\phi(y) \\ \psi^3(x, y) &= \psi(x)\psi(y)\end{aligned}\tag{2.12}$$

Just as in the one dimensional case, the wavelet decomposition of a two dimensional signal involves convolving the signal with the separable, two dimensional scaling function and three wavelet functions given by equation 2.11. Just as the exact form of the scaling and wavelet functions in the one dimensional case was not required for the decomposition and the QMFs with impulse response $h(n)$ and $g(n)$ were utilized, the two dimensional QMFs filter coefficients are generated as follows :-

$$\begin{aligned}h_{LL}(k, l) &= h(k)h(l) \\ h_{LH}(k, l) &= h(k)g(l) \\ h_{HL}(k, l) &= g(k)h(l) \\ h_{HH}(k, l) &= g(k)g(l)\end{aligned}\tag{2.13}$$

The two dimensional wavelet decomposition and reconstruction are illustrated in the Figures 2.3 and 2.4 respectively.

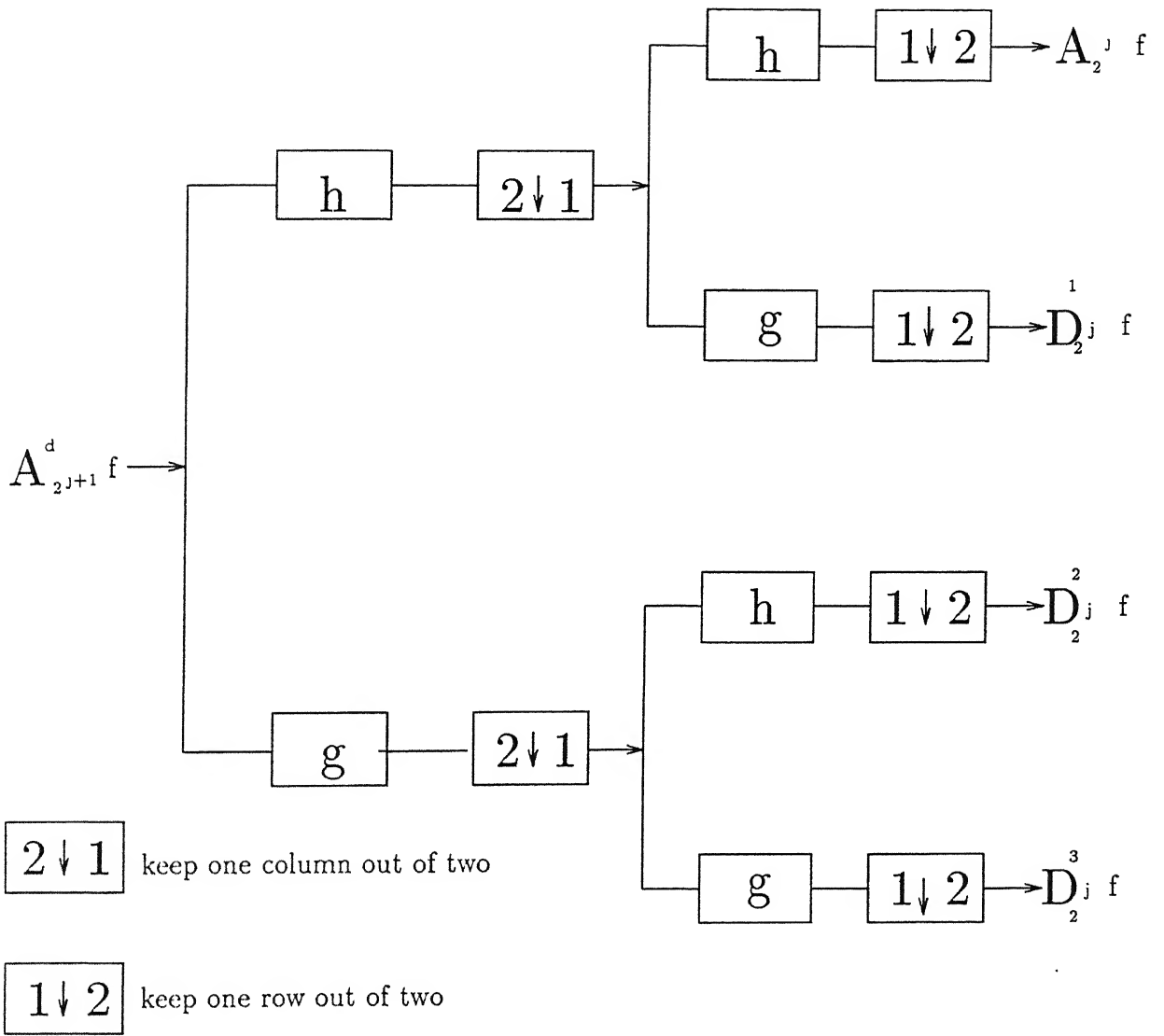


Figure 2.3: 2D Wavelet Decomposition

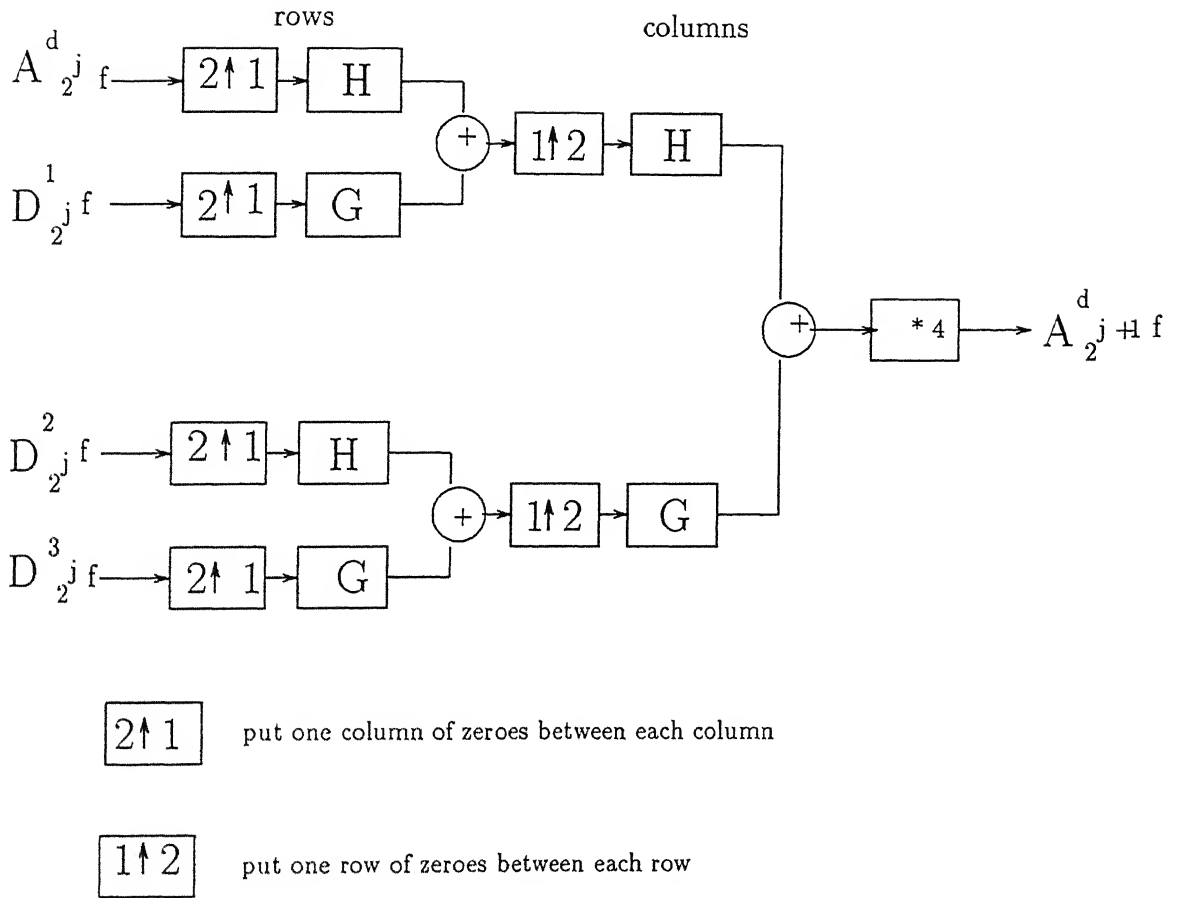


Figure 2.4: 2D Wavelet Reconstruction

2.5 Wavelet Packet Decomposition

As a natural extension to the wavelet transform theory, Coifman, Meyer and Wick-erhauser first proposed a generalization of wavelet theory referred to as Wavelet Packet. It is well established that wavelet packets define efficient schemes for representing and compressing images. The idea is to construct a library of orthonormal bases for function $f(x) \in L^2(R)$ by generalizing the method of multiresolution decomposition. The library of orthonormal bases also has time-frequency localization properties which can be reasonably well controlled. In the 1-D case the library construction is carried out by considering a quadrature mirror filter $h(n)$. Define a summable sequence $h = h_n$ satisfying

$$\sum_n h_{n-2k} h_{n-2l} = \delta_{k,l}, \sum_n h_n = \sqrt{2} \quad (2.14)$$

and let $g = g_k$ where $g_k = (-1)^k h_{1-k}$. Introducing a summing and differencing operators denoted by F_0, iF_1 respectively. we denote :

$$F_0 s_k(2i) = \sum_k s_k h_{k-2i} \quad (2.15)$$

and

$$F_1 s_k(2i) = \sum_k s_k g_{k-2i} \quad (2.16)$$

The adjoints F_0^*, F_1^* satisfy

$$F_0^* F_0 + F_1^* F_1 = I, F_0 F_0^* = F_1 F_1^* = I, F_0 F_1^* = F_1 F_0^* = 0 \quad (2.17)$$

where the mapping F , defined by $F = F_0 \oplus F_1$ is orthogonal. These two summing and differencing operators are applied to split the functional space. The Wavelet transform decomposition is obtained by splitting V_{j-1} with these operators into V_j and W_j and doing the same recursively for V_j . In the frequency domain, the splitting operation corresponds to dividing the frequency interval into both the low as well as high frequency parts. Hence the wavelet packets allow more flexibility in adapting the basis to the time and frequency contents of a function. Thus the decomposition proceeds as a binary tree which is illustrated in the Figure 2.5. Similarly for 2D functions, the analysis functions are given by equation 2.12. The wavelet

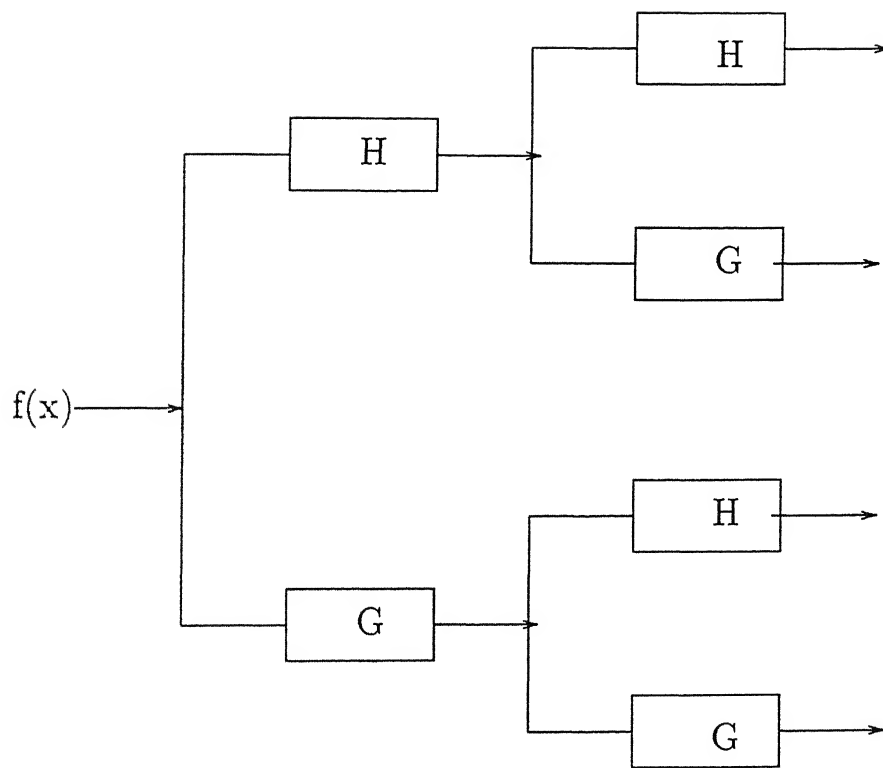


Figure 2.5: 1D Wavelet Packet Decomposition

packet decomposition of a 2D function proceeds as a quad tree, as the decomposition is applied to the coarse signal as well as the three detail signals at each stage respectively.

Similar to the summing and differencing operators in the one dimensional case, we can define four two-dimensional convolution-decimation operators in terms of the QMFs H and G , namely the tensor products of the pair of QMFs as under:

$$\begin{aligned} F_0 &= H \otimes H \\ F_1 &= H \otimes G \\ F_2 &= G \otimes H \\ F_3 &= G \otimes G \end{aligned} \tag{2.18}$$

$$\tag{2.19}$$

The two dimensional wavelet packet decomposition in terms of the convolution decimation operators proceeds as a quad tree as illustrated in the Figure 2.6. The tiling diagrams illustrating the various subspaces is shown in Figures 2.7 and 2.8.

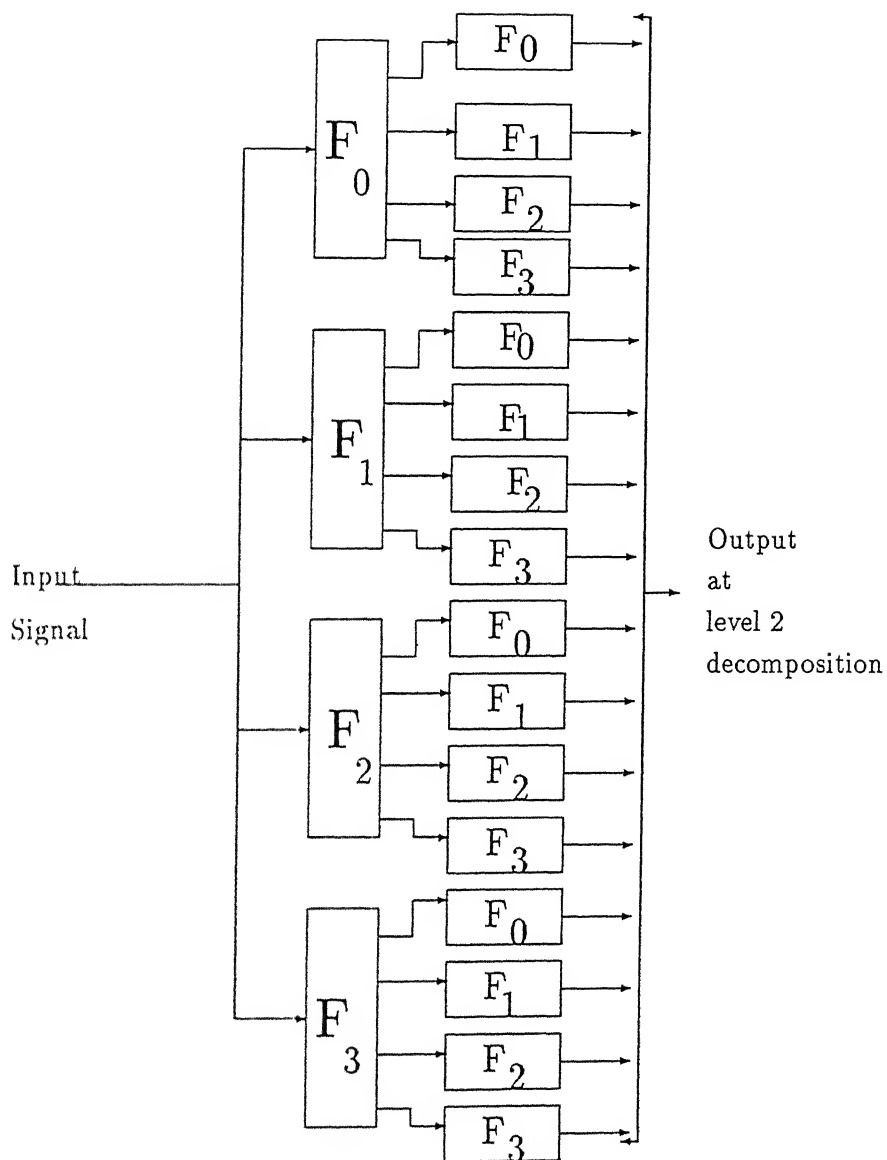


Figure 2.6: 2D Wavelet packet decomposition

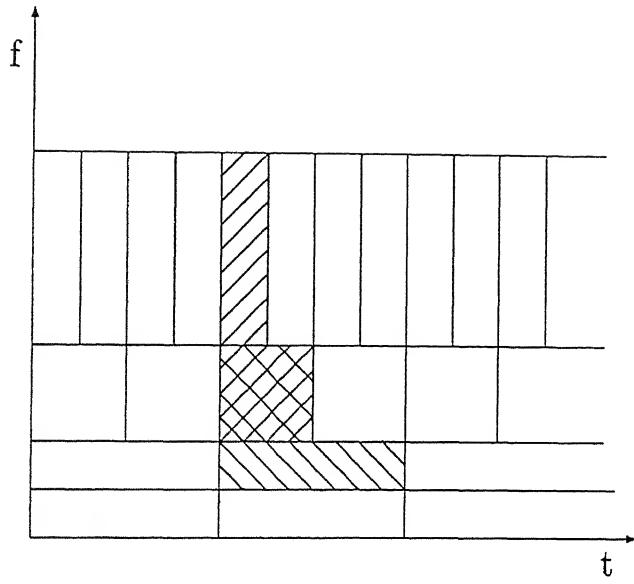


Figure 2.7: Wavelet basis

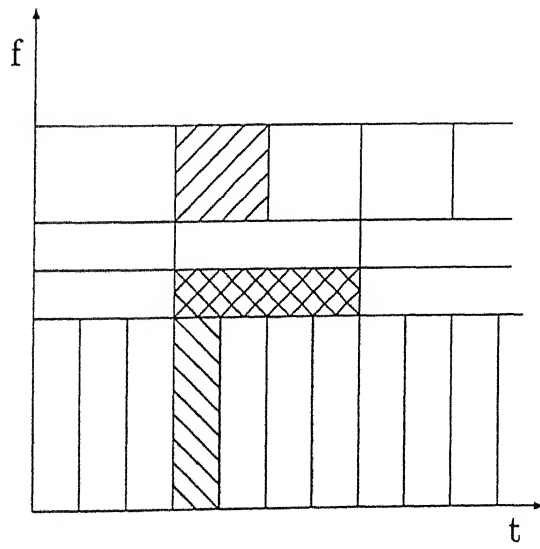


Figure 2.8: Wavelet packet basis

Chapter 3

Texture Classification And Region Identification

3.1 Concept of Wavelet Packet Signature

Different textures in an image normally have different power spectral signatures. In a wavelet packet decomposition the basis functions are obtained by a translation and scale change of the wavelet and scaling functions. Since they are orthogonal they remain well localized in scale and spatial domain. The complete wavelet packet tree represents the distribution of the signal in the scale space domain. The total number of coefficients in a complete decomposition is exactly equal to the number of points in the original signal. Since the wavelet packet form an orthogonal basis their decompositions preserve the energy of the signal at each level. Hence a decomposition at any level in the tree of the wavelet packets will have the same energy as the signal. Thus if the energy of the signal is $E_{0,0}$ then the decomposition at level k will have the energy distribution as

$$E_{0,0} = \sum_{l=0}^{4^k-1} E_{k,l} \quad (3.1)$$

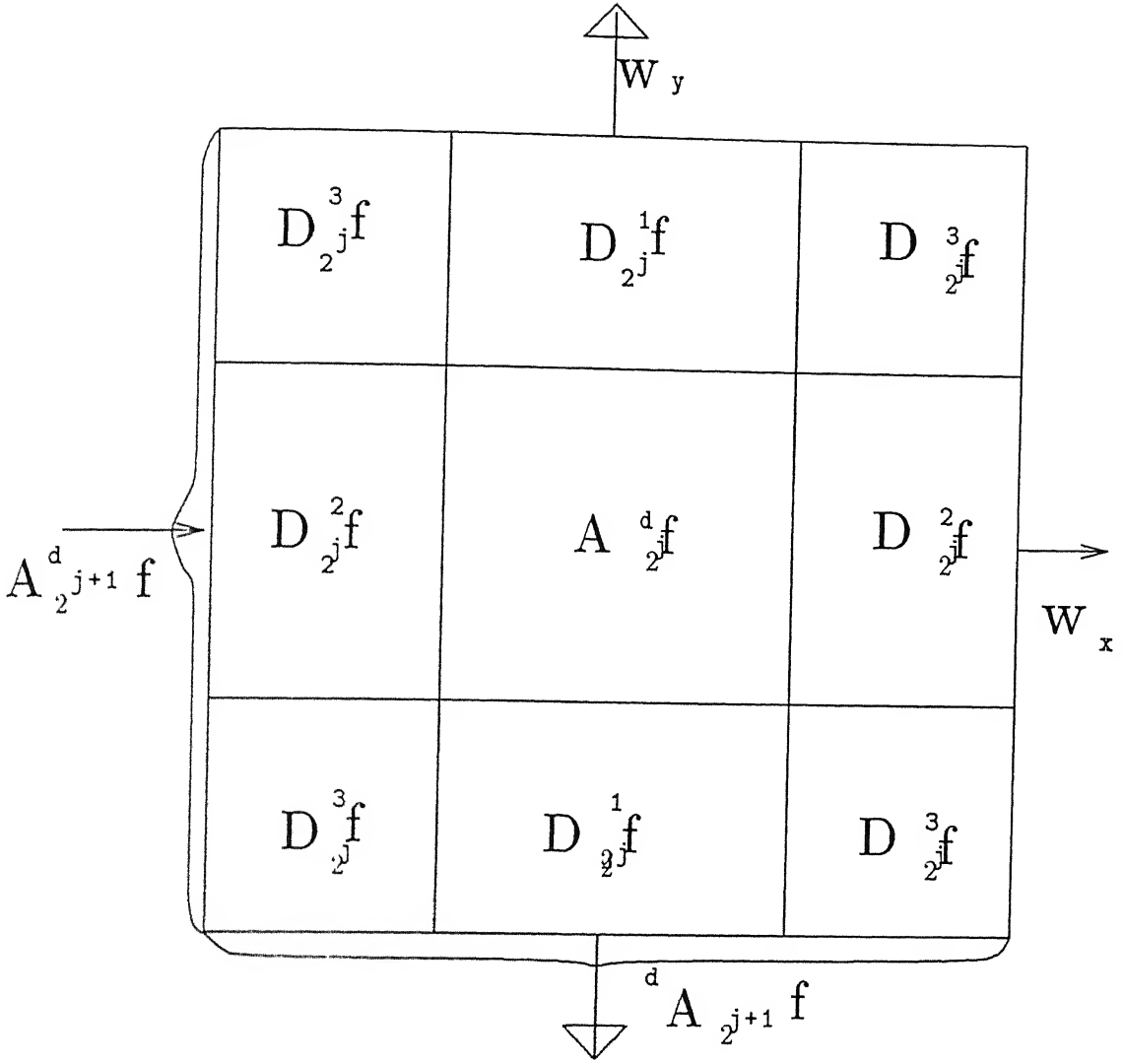


Figure 3.1: Frequency plane division of the wavelet transform

Thus the energy of the signal is distributed amongst the various subspaces depending on the frequency content of the signal. Since the 2D wavelets capture the orientation information in the various subspaces the energy distribution forms a unique identification of the signal. The division of the frequency plane at the first level of decomposition of the wavelet transform is displayed in fig 3.1. Hence a feature vector consisting of a set of energy values obtained from the wavelet packet decomposition forms a unique identifying signature of the signal and supports a representation for signal classification. For 2D signals the wavelet packet energy signature is obtained as an extension of 1D wavelet packets. In 2D case, the energy of the signal at any

	Haar	C-6	D-4	D-8	D-20
Variance of ϕ	0.0832	0.0986	0.1416	0.179	0.2192

Table 3.1: Variance of Scaling functions

node is distributed between four children nodes as given by the equation

$$E_{n,m}^p = E_{2n,2m}^{p-1} + E_{2n,2m+1}^{p-1} + E_{2n+1,2m}^{p-1} + E_{2n+1,2m+1}^{p-1} \quad (3.2)$$

Since orientation selectivity is obtained by the tensor product of the low pass filter h and high pass filter g , the energy distributions are computed for the three different orientations. Figure 3.7 shows the set of frequency channels associated with wavelet packet transforms for level one and two.

3.2 Selection of Filters

To capture the energy of the signal in the various subspaces, it is desirable that the wavelets used have higher energy concentrations. Since the wavelet function is related to the scaling function they would share the same kind of property regarding the energy concentration. Hence the study is confined to the scaling function which is related to the low pass filter $H(z)$. The energy concentrations of the scaling function and the localization of ϕ are determined by the spatial variance of the scaling function.

3.3 Spatial Characterization of the Scaling Function

The spatial variance of the scaling function is studied by the spatial characterization of the scaling function, which determines the evolution of $\phi(x)$ with respect to x .

The criterion m_k is introduced to characterize the scaling function ϕ spatially. To define m_k , the function $p(x)$ is introduced such that

$$p(x) = \frac{|\phi(x)|^2}{\int |\phi(x)|^2 dx} \text{ with } p(x) \geq 0 \text{ and } \int p(x)dx = 1 \quad (3.3)$$

m_k is then defined as

$$m_k = \int (x - m_1)^k p(x)dx \text{ with } m_1 = \int xp(x)dx \quad (3.4)$$

Thus m_1 is equal to the mathematical expectation of $p(x)$ and m_2 corresponds to the spatial variance of the scaling function ϕ . The quantity m_2 allows us to determine the energy concentration of ϕ and provides information on the spatial length or localization of ϕ . This criterion also applies to the wavelet ψ . The Table (3.1) has the calculations for the spatial variance of various scaling functions. It is observed that the Daubechies filters have highest spatial variance and thus have better energy concentrations. Hence these filters are better suited for texture discrimination based on the energy criterion.

3.4 Minimum Distance Classification

To test the effectiveness of the wavelet packet signatures for texture classification the minimum distance classifier was used . The minimum distance classifier was based on the assumption that each type of texture is represented by a prototype of the texture. This texture is represented by the energy distribution across the wavelet packet subspaces. The unknown texture is assigned to this texture class when the Euclidean distance D_k is the minimum. The Euclidean distance is defined by the equation

$$D_k = \|X - Z_k\| = \sqrt{\sum_{j=0}^{N-1} (x_j - z_{j,k})^2}. \quad (3.5)$$

X represents the feature vector of the unknown texture and $Z_k \forall k$ represents the set of prototypes of the various textures. The prototype of each class is estimated by using the mean of the training sample given by

$$J_k = \sum_{X \in w_k} \|X - Z_k\|^2, k = 1, 2, \dots, M. \quad (3.6)$$

where M is the number of texture classes.

3.5 Classification Algorithm

There are two approaches to texture classification. One is the progressive classification method [[5]] and other is the method of classification using the energies of all the subspaces at a given level as feature vectors.

3.5.1 Progressive Classification

Learning Phase

- (1) Generate m samples from the given image. Take the image of size ' $l \times l$ ' and a window of size ' $n \times n$ '. Move the window over the entire image in steps of ' k ' to get desired number of samples.
- (2) Take the first sample and decompose this sample using the wavelet packet transform. Calculate the normalized energy ' e ' of all the children by the equation given below:

$$e = \frac{1}{MN} \sum x_{m,n}^2, 0 < m < M, 0 < n < N \quad (3.7)$$

- (3) Continue till the "stopping criterion" is reached. The stopping criterion is the smallest size of the subspace. Usually this is a 16×16 subspace.
- (4) Repeat this process for all the ' m ' samples of the image.
- (5) Average the energy maps over all the m samples and generate the representative an energy map for this image.
- (6) In the representative energy map, at each level of the decomposition discard those subspaces which have significantly lower energy compared to the other subspaces.
- (7) Repeat the steps for all the images and build a energy map data bank for classification.

Classification phase

- (1) Generate the energy map of the unknown image in the above manner.
- (2) Pick up the dominant subspaces at a given level and arrange them in the descending order.
- (3) Arrange energy of the images in the the data bank also in a similar manner.
- (4) Discard those images which do not have the similar mapping.
- (5) Calculate the distance measure of the unknown image with the remaining images and assign the unknown image to the image with the lowest distance .

The algorithm is given in the flow chart in the Figure 3.2 and 3.3.

3.5.2 Classification Using Complete Energy Map

In the previous algorithm we did not utilise all the subspaces at the given level. Only the dominant subspaces were chosen and their energy values were utilised for classification. We had assumed that high energy implies a better discriminability. This may be true with those images which have their energy concentrated in a few distinct subspaces. However a large class of textures exist which have the distribution of the signal energy in all the subspaces in a similar manner. Also if a large number of textures have the similar energy distribution, more number of features will be required for classification. This problem can be solved by using all the subspaces at the lowest level of decomposition for building the feature vector. Hence another algorithm is proposed, utilising the complete energy. This also has two phases as in the progressive classification algorithm..

Training Phase

- (1) Generate ' m ' samples from the given image as explained in subsection 3.5.1.
- (2) Take the wavelet packet decomposition of one sample of the image for all levels till the smallest size of the subspace, after decomposition, is 16×16 .
- (3) Calculate the normalized energy of all the subspaces at the lowest level by equation 3.7.

TRAINING PHASE

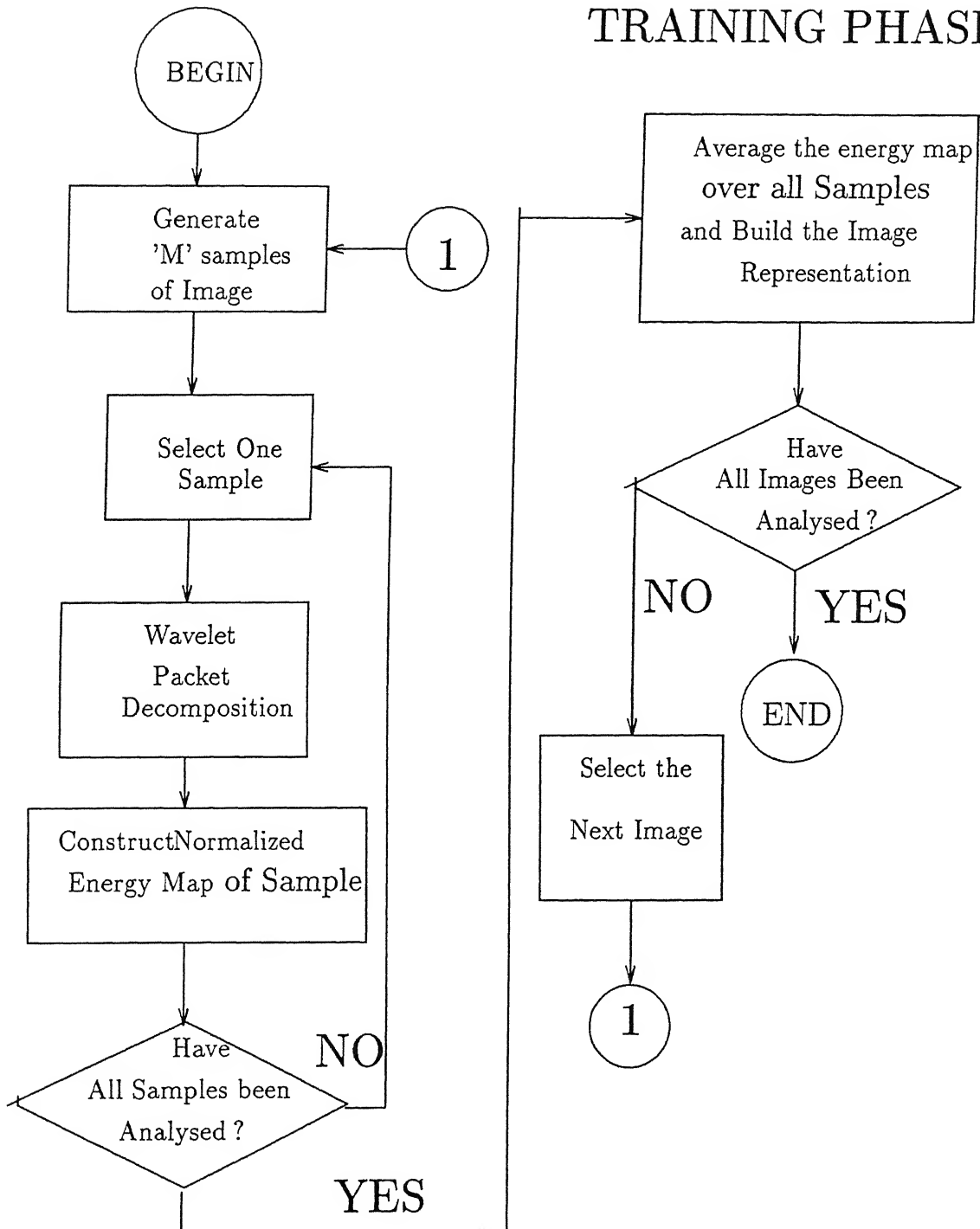
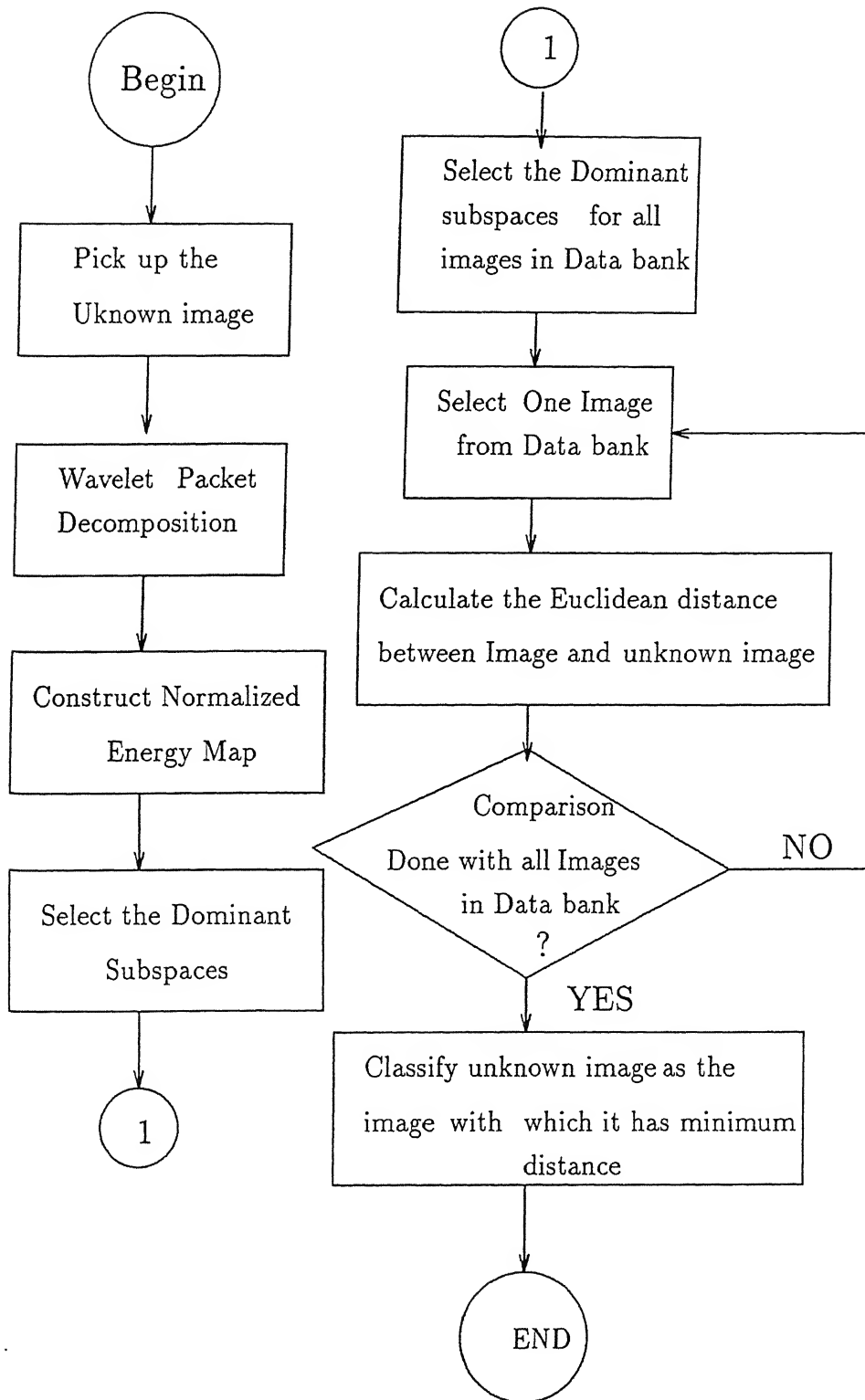


Figure 3.2: Flow Chart for Training Phase of Progressive Classification



CLASSIFICATION PHASE

Figure 3.3: Flow Chart for Classification Phase of Progressive Classification

- (4) Repeat this process for the set of given samples and generate the representative energy map by averaging the energy map over all the samples. This energy map is the feature vector for the given image.
- (5) Build the feature vector for all the given images in a similar manner.

Classification Phase

- (1) Take the unknown image and calculate its wavelet packet decomposition.
- (2) Form the feature vector of this image by calculating the energy of all the subspaces at the lowest level.
- (3) Calculate the distance for this image with all the given images in the data bank using the minimum distance classifier in equation 3.5.
- (4) The image is assigned the class with which it has the minimum distance.

The complete energy map classification is explained in the flow chart in Figure 3.4 and 3.5

3.6 Implementation for Single Textured Images

In a large no of references dealing with the texture analysis [5],[10],[13],[18], images were chosen from the Brodatz album of Textures. Due to the non availability of the album, synthetic textures with the desired orientations were generated. Since the algorithm proposed is for textures with distinct directional orientations, the generation of synthetic textures gives the freedom to generate the textures with the orientations which can be captured by the wavelet bases. A set of ten images were synthesised with distinct directional orientations. Then a set of 65 samples were taken from each image, by selecting different regions from the image. The procedure followed is is similar to that used in Multispectral image processing. In the present method Fifty samples per image were utilized for building the class prototypes, in the Training phase. The fifteen samples were used as the test images to implement the classification phase. Both the algorithms were implemented. The

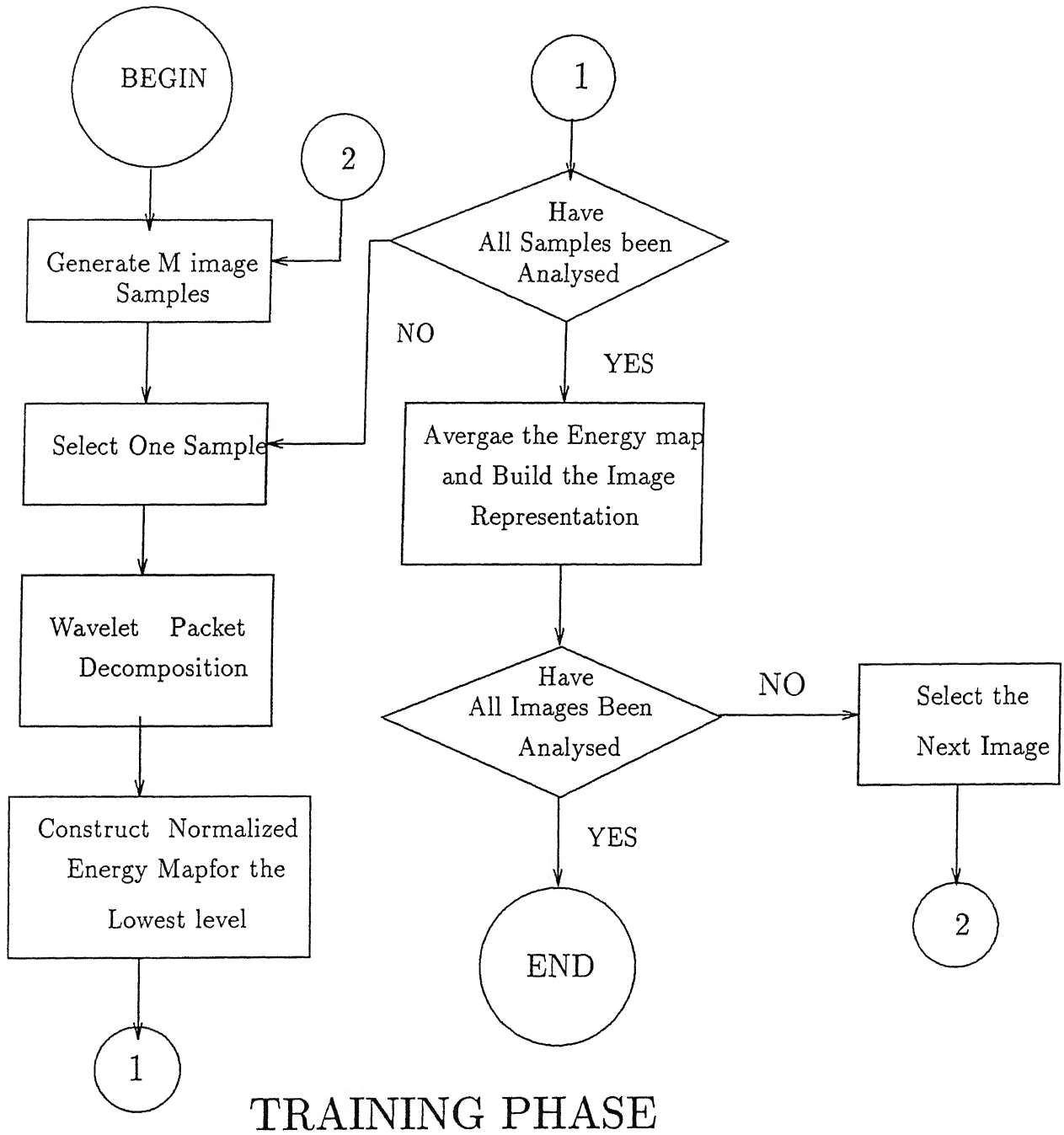
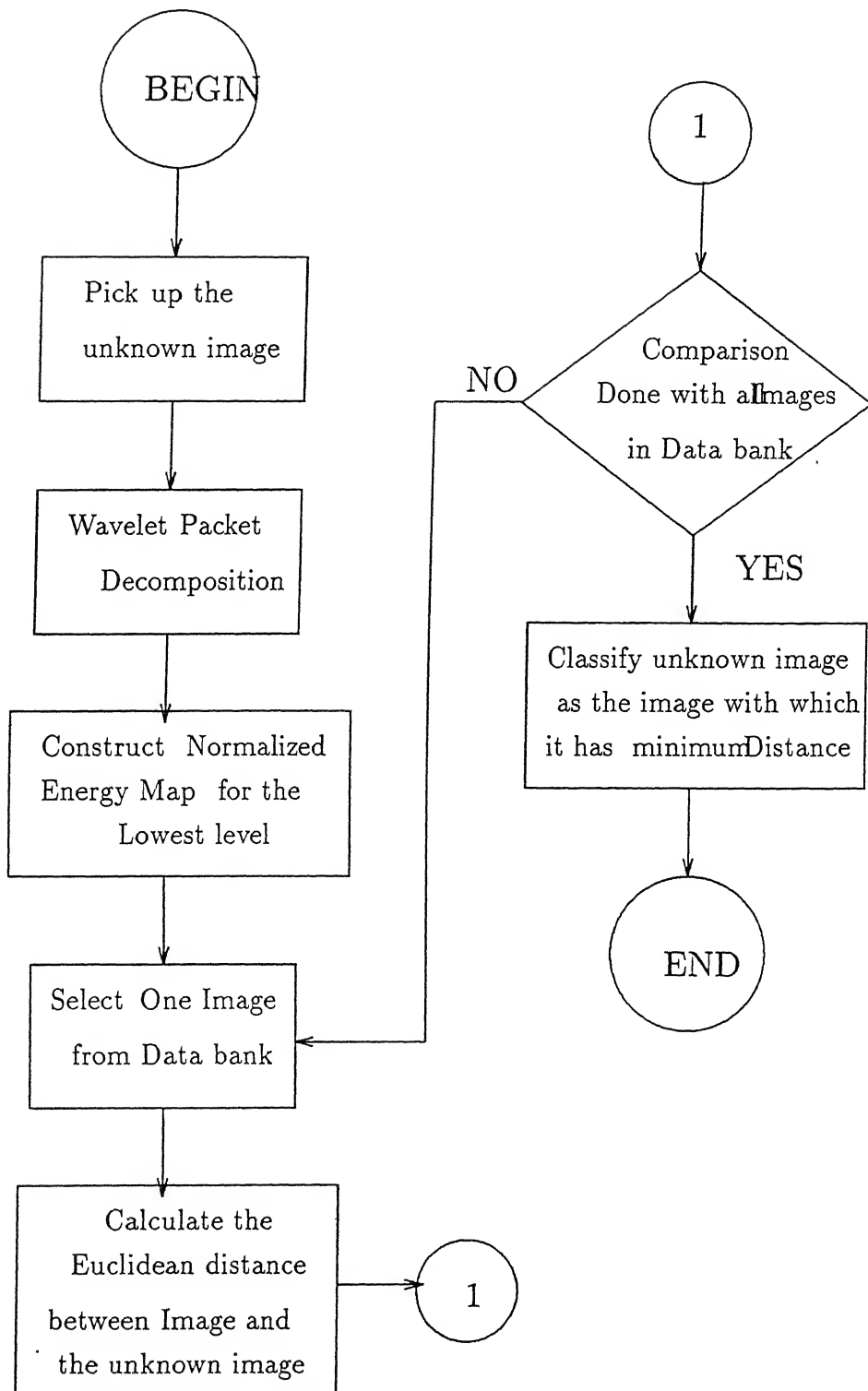


Figure 3.4: Flow Chart for Complete Energy Map Classification



CLASSIFICATION PHASE

Figure 3.5: Flow Chart for Classification Phase of Complete Energy Map Classification

$h(0)$.48296291
$h(1)$.8365163
$h(2)$.22414386
$h(3)$	-.129409599

Table 3.2: Daubechies 4-tap filter wavelet coefficients

implementation was done using the Daubechies 20-tap and 4 tap filter. The wavelet and scaling functions of both these filters are displayed in Figures 3.8, 3.9, 3.10, 3.11 respectively. The coefficients are given in table 3.2 and 3.3. The energy plots for all the image prototypes are given in Figures 3.12 to 3.21.

3.6.1 Results

Having implemented the progressive classification and complete energy map classification algorithms for single textured images, following are the observations.

- (1) Since the progressive classification algorithm uses only the dominant subspaces for classification it had a larger error of classification when the number of feature vectors were few. For only two or three selected dominant subspaces it gave very poor results. As the number of features in the feature vector were increased the classification rate improved. This is displayed in the plot in the Figures no 3.22 and 3.23 Though the Daubechies 4-tap filter is only marginally inferior to the 20-tap filter, it needs analysis with a larger no of texture samples, to come to a definite conclusion. However ,since the Daubechies 20 tap filter has a higher energy concentration, it will have better discriminability for textures using the energy criterion.
- (2) The algorithm using the complete energy map gave classification errors at level one and two. But at level 3 with 64 features and level 4 with 256 features 100

$h(0)$	0.0266700679005473
$h(1)$	0.1881768000776347
$h(2)$	0.5272011889315757
$h(3)$	0.6884590394534363
$h(4)$	0.2811723436605715
$h(5)$	-.2498464243271598
$h(6)$	-.1959462743772862
$h(7)$	0.1273693403357541
$h(8)$	0.0930573646035547
$h(9)$	-.0713941471663501
$h(10)$	-.0294575368218399
$h(11)$	0.0332126740593612
$h(12)$	0.0000000000000070
$h(13)$	-.0107331754833007
$h(14)$	0.0013953517470688
$h(15)$	0.0019924052951925
$h(16)$	-.0006858566949564
$h(17)$	-.0001164668551285
$h(18)$	0.0000935886703202
$h(19)$	-.0000132642028945

Table 3.3: Daubechies 20-tap filter wavelet coefficients

percent classification was obtained. Also though the number of features are more, computationally it is not time consuming. Hence we can derive 100 % classification using this method.

- (3) The wavelet transform decomposition is inferior in classification to the wavelet packet transform because a fewer no of features are available at any level of decomposition compared to the wavelet packet decomposition. Also the images with same energy in the higher frequencies will have the same energy map with the wavelet transforms. Hence these images will not be discriminated correctly.

3.7 Region identification and Classification

In many of the segmentation techniques, the textured images are differentiated into regions of different textures using various techniques both spatial and spectral. However the region identification and classification does not take place. A technique to carry out region identification and classification is proposed using the wavelet packet transforms. The wavelet packet decomposition decomposes the image across the scale, while preserving the spatial locations of different regions. Also the energy in each subspace at every level is obtained from the contributions of the energies of the textures present in the image. Each texture contributes the energy in those subspaces where its orientations are represented. Thus, if the dominant subspaces of a particular texture presented in the image are selected from the data bank and the image is reconstructed from its decomposition, with the chosen subspaces only, the region containing that texture is reconstructed and the rest of the image is suppressed. Subsequently to classify the region the reconstructed region is extracted and classified using the minimum distance measure with the stored textures as discussed in section 3.5. The procedure for region identification is enunciated.

- (1) Build the data bank of energy distributions of various textures after taking the wavelet packet decomposition of the images.
- (2) Take the wavelet packet decomposition of the given image.

- (3) Choose a texture from the databank and identify its dominant subspaces using the energy criterion.
- (4) Having chosen the dominant subspaces at a given level of decomposition for the chosen texture from the data bank, these subspaces are used to reconstruct the image from its decomposition.
- (5) Extract the reconstructed region of the image and carry out classification of this region.
- (6) If the image is classified incorrectly then repeat the procedure
- (7) Check the classification once again.

This is enunciated in Figure 3.6.

3.8 Image Rescaling

To enhance the extracted region the region is rescaled by rescaling the wavelet packet coefficients at the corresponding spatial locations at the desired resolution. For the purpose of explanation we will assume that the picture is a function of two variables supported in the region $[0,1] \times [0,1]$. Let (n,m,k) be the index of an amplitude in the complete wavelet packet expansion of a picture S . Here $m=0,1,\dots,l$ i.e. the no of levels and $0 \leq n < 4^m$. We rescale the image by replacing $c(n,m,k)$ with $c(n',m',k')$ for a restricted range of k 's. The map n to n' etc determined by the rescaling. For example, if $n'=2n$ and $m'=m+1$, then we will increase the magnification locally with little change in the frequency content.

3.9 Results and Discussion

The process of region identification and classification was carried out with two textured and three textured images with distinct directional orientations. Since separable wavelets were used selectivity in three directions could be obtained because each

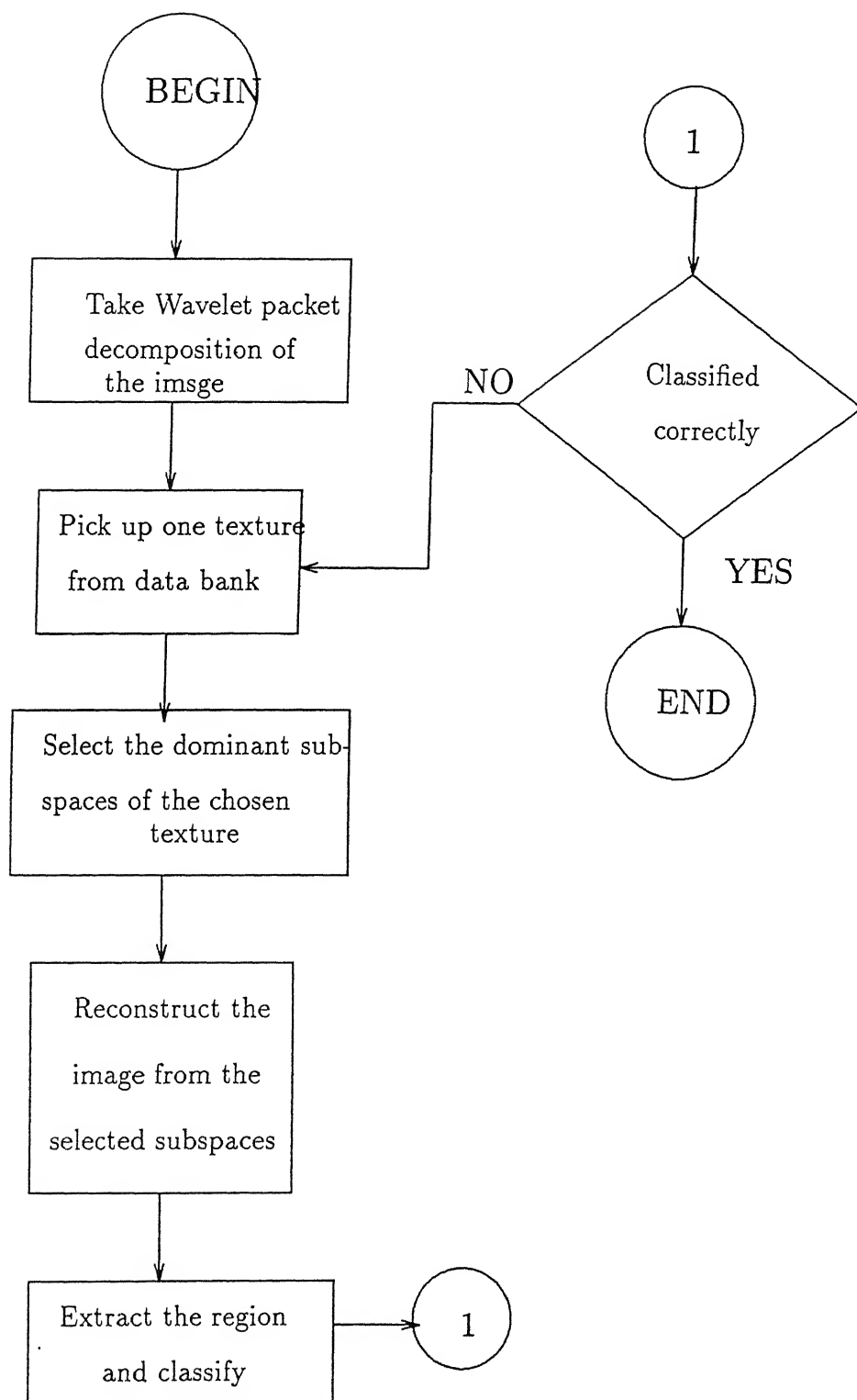


Figure 3.6: Flow diagram region identification

detail image corresponds to directional edges in horizontal, vertical and diagonal directions. The various observations are

- (1) In two textured images since there were two partitions with distinct directions, the segmentation and reconstruction was distinct.
- (2) Though only the dominant subspaces are chosen, the reconstructed portion of the desired image was classified correctly. Though the resolution of the reconstructed portion of the image is poorer, compared to the original image, it is still adequate for correctly identifying the different regions.
- (3) In the three textured images also the regions were correctly classified. To ensure that the undesired portion of the image was not reconstructed the dominant subspaces common to the undesired texture were dropped and region identification and classification was then carried out. This process has to be done judiciously, since dropping of the common subspaces may result in the desired portion of the image losing a large amount of its details.
- (4) The results of the above described method are shown in the Figure 3.24 to 3.27.
- (5) Though we concentrated on distinct orientations, this algorithm can be used for arbitrary textures also. Since we used orthonormal wavelet bases, choice of the type of wavelets, does not influence the algorithm, for region identification

3.10 Highlights of The Proposed Method

It is observed that the wavelet packet transform provides a good analytic tool for texture analysis. Till now in the multiresolution analysis of textures wavelet transforms or Gabor Transforms were used, which were suitable for textures with energy concentrated in the low frequency. The wavelet packet transform is more suitable and effective for textures with dominant middle frequency channels. In the multiresolution analysis the features are extracted using the transforms and subsequently feature vector analysis is done using KL transforms for feature space reduction.

These methods either carry out segmentation or classification. In the proposed method a combination of the two has been attempted with a simple algorithm. Since most of the textures energy characteristics would be known apriori , this method is simple. With more and more powerful computational techniques being now available, implementation of the proposed algorithm will not be difficult.

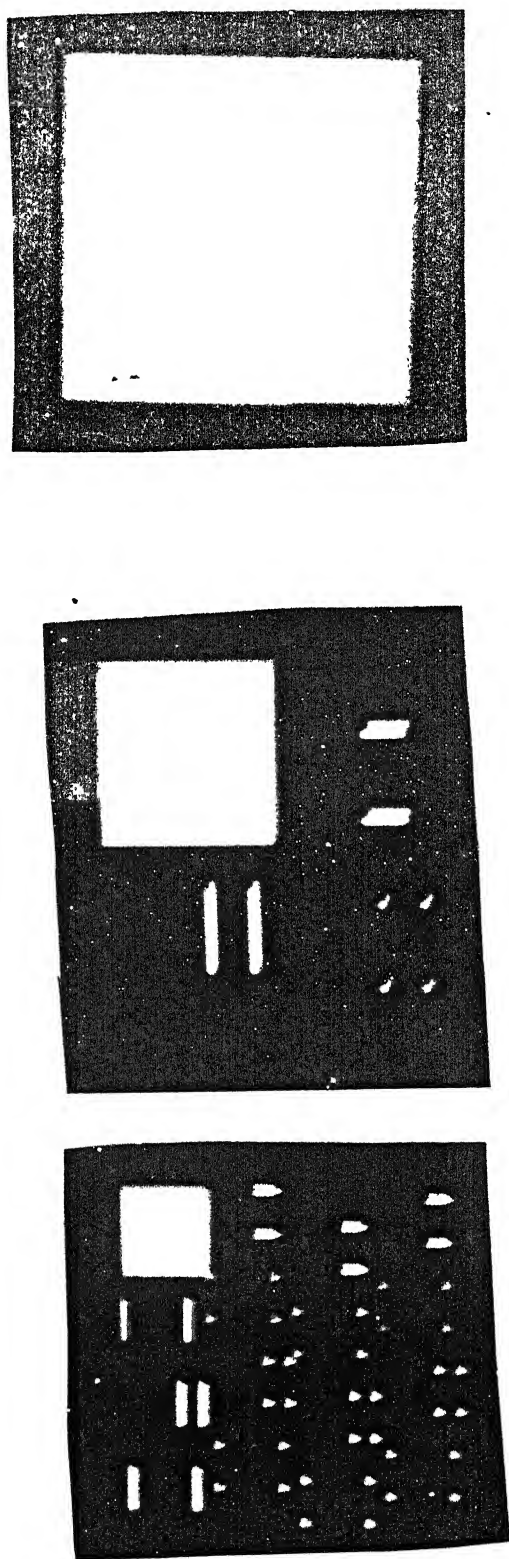


Figure 3.7: Display of wavelet coefficients at level1 and level2

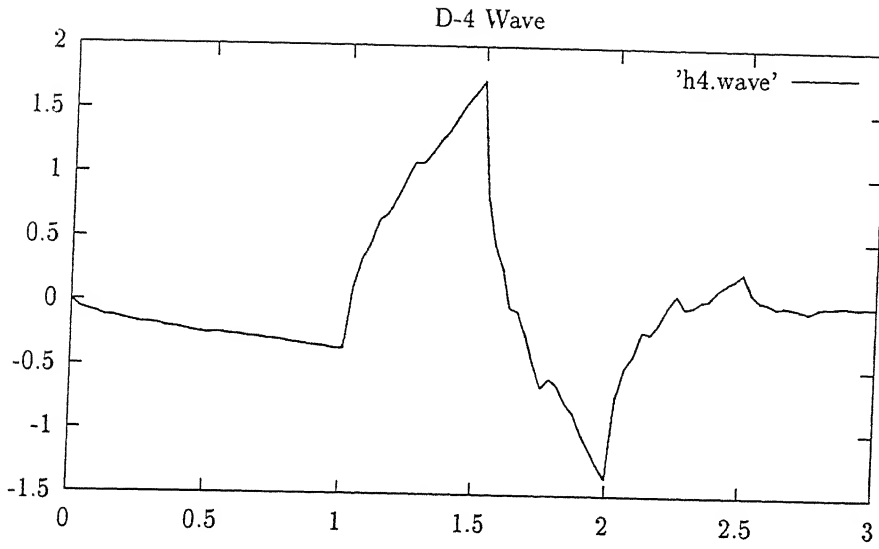


Figure 3.8: Daubechies 4-tap Wavelet function

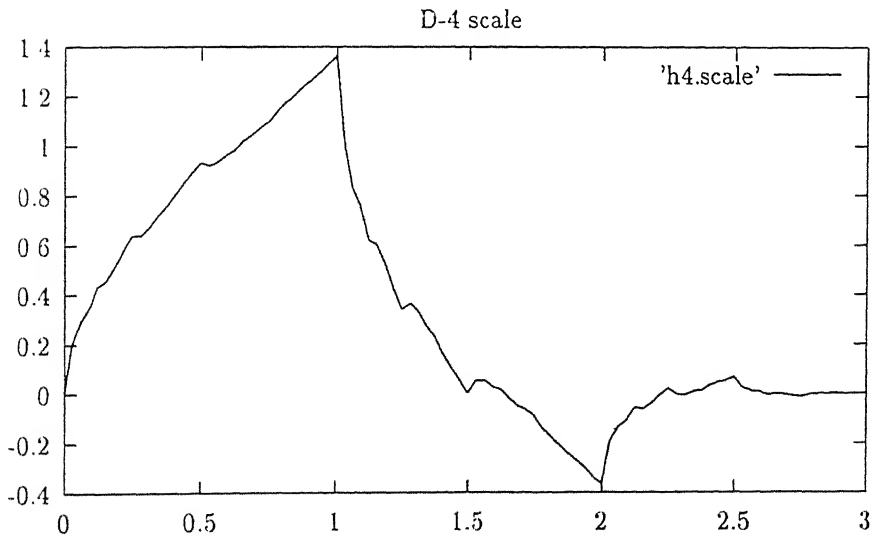


Figure 3.9: Daubechies 4-tap Scaling function

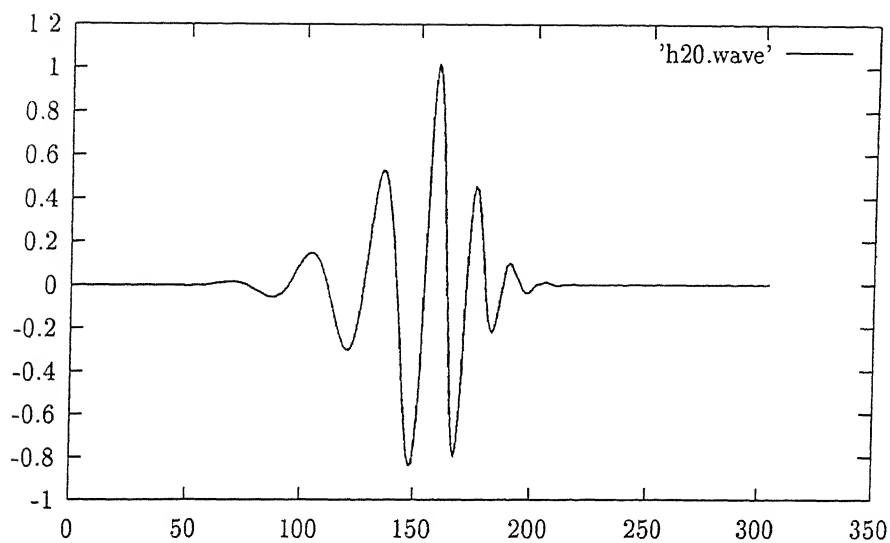


Figure 3.10: Daubechies 20-tap Wavelet function

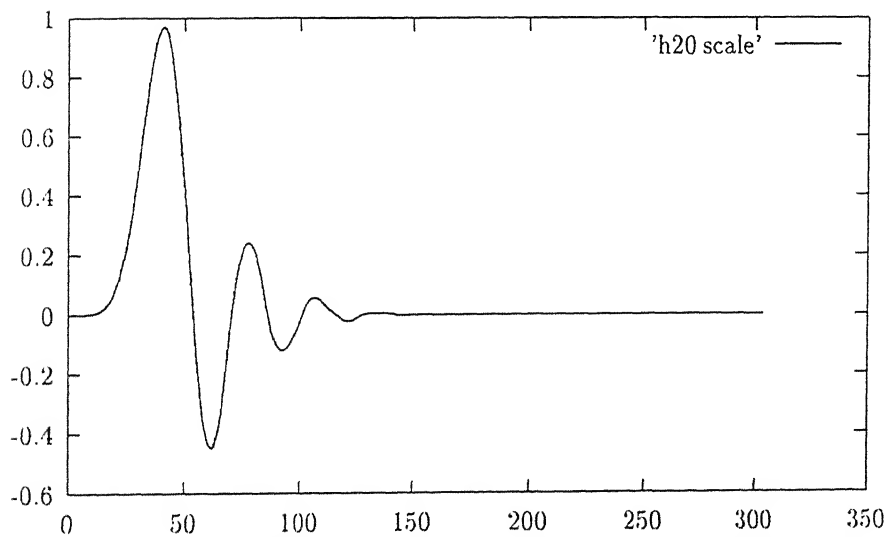


Figure 3.11: Daubechies 20-tap Scaling function

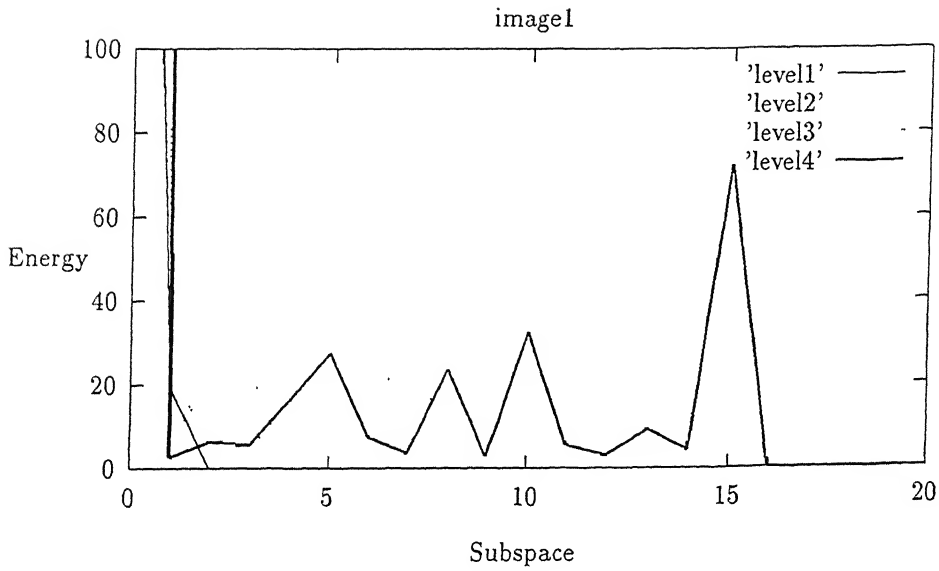


Figure 3.12: Energy distribution Image1

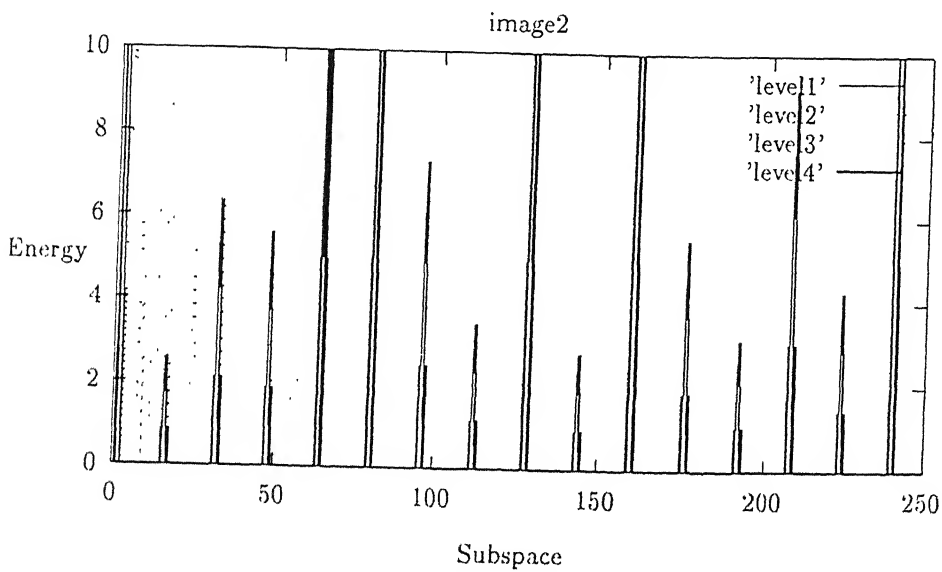


Figure 3.13: Energy distribution Image2

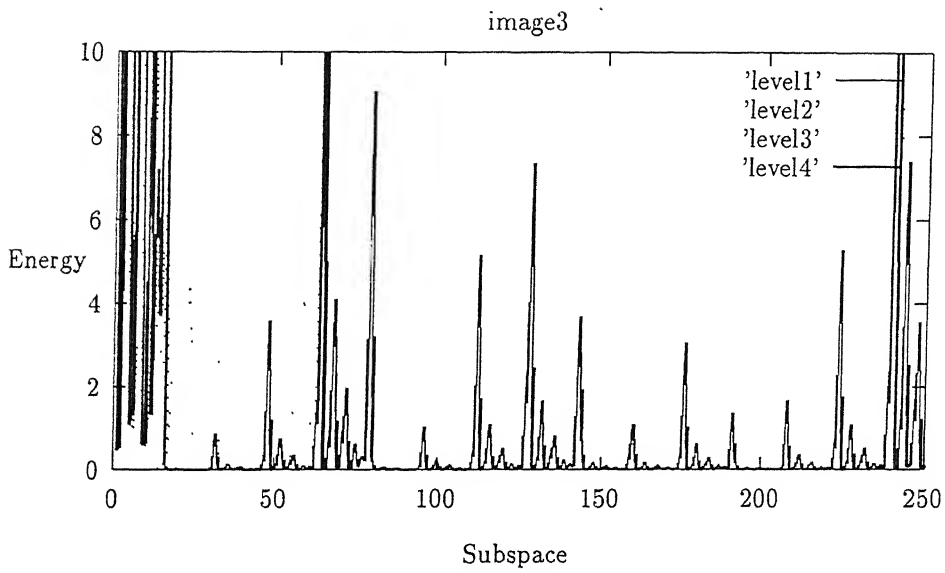


Figure 3.14: Energy distribution Image3

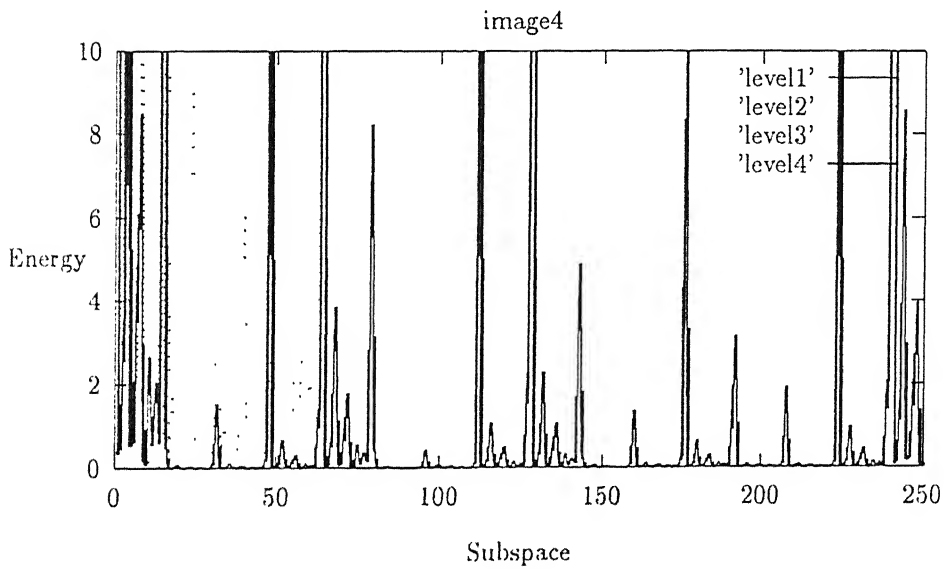


Figure 3.15: Energy distribution Image4

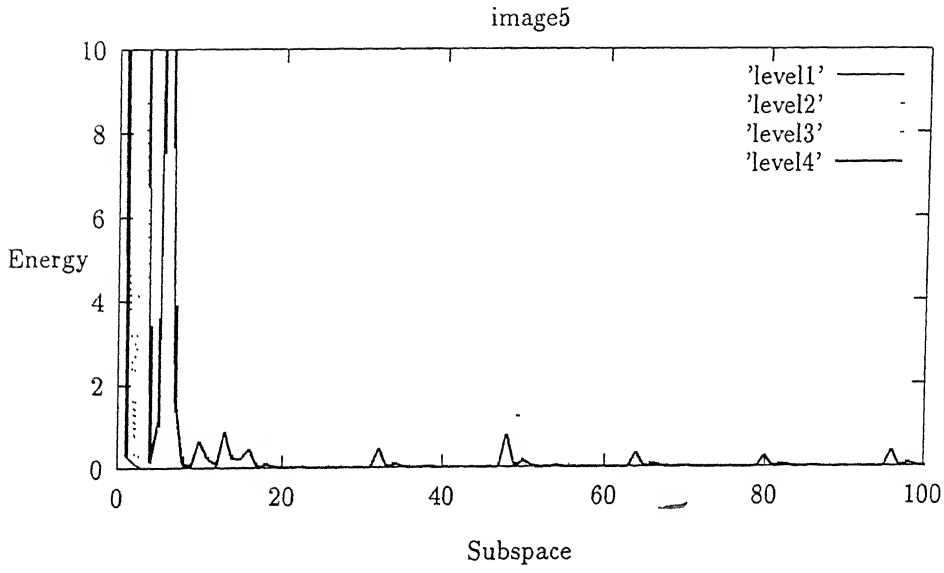


Figure 3.16: Energy distribution Image5

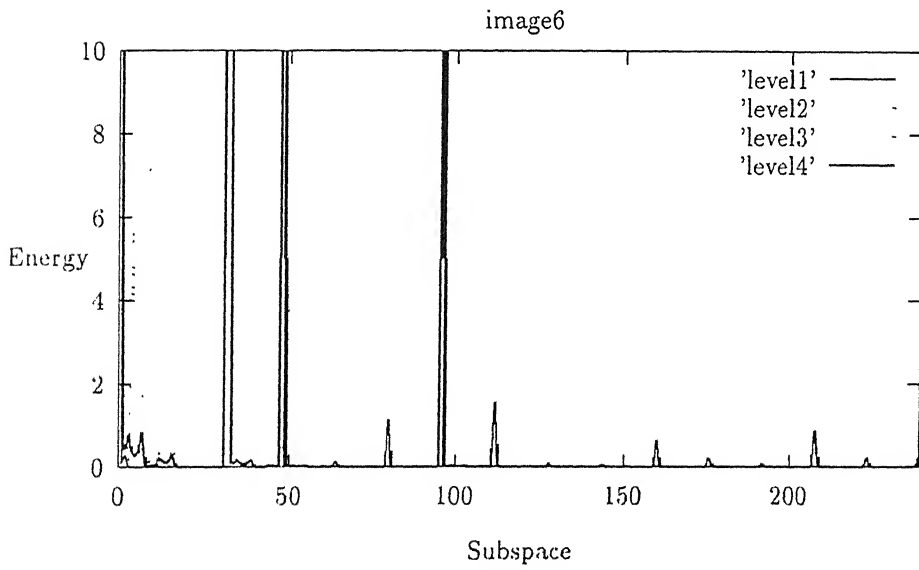


Figure 3.17: Energy distribution Image6

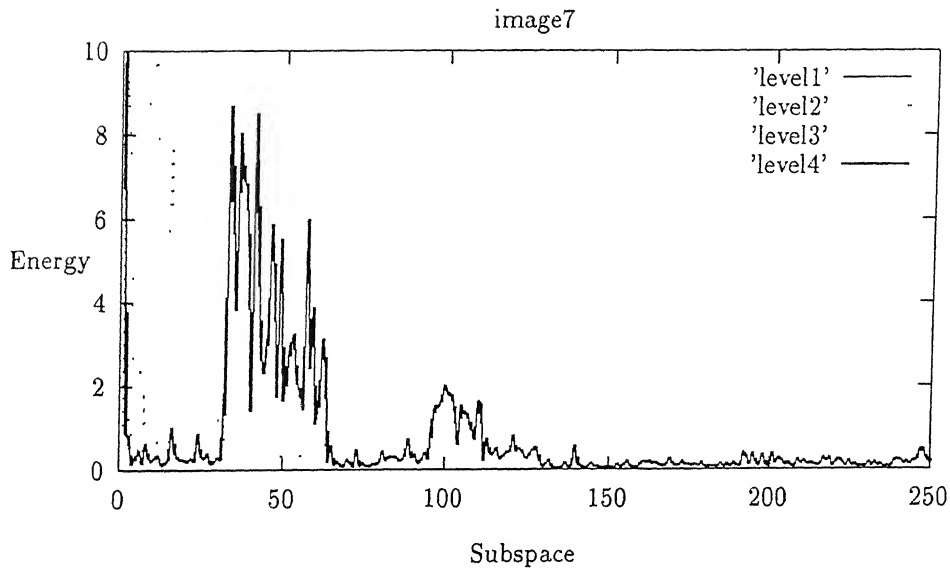


Figure 3.18: Energy distribution Image7

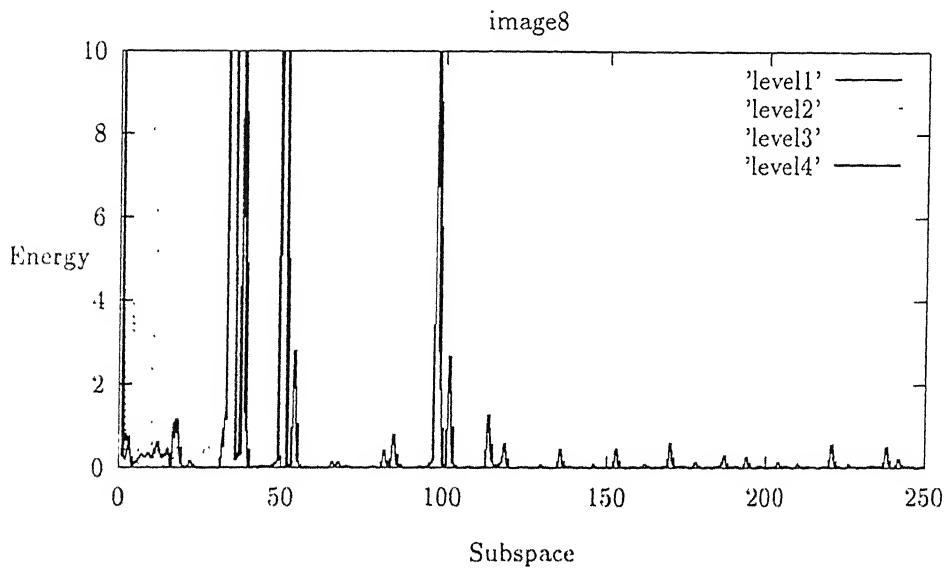


Figure 3.19: Energy distribution Image8

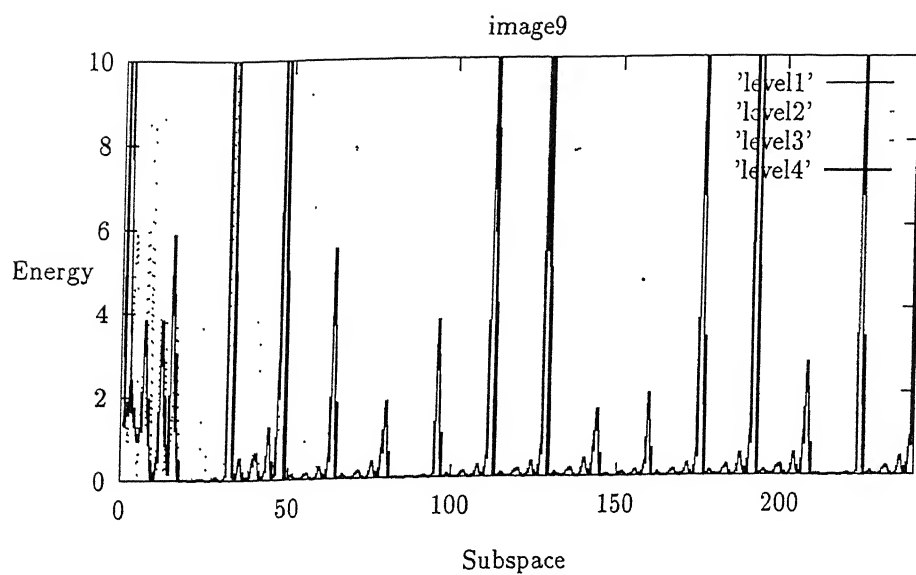


Figure 3.20: Energy distribution Image9

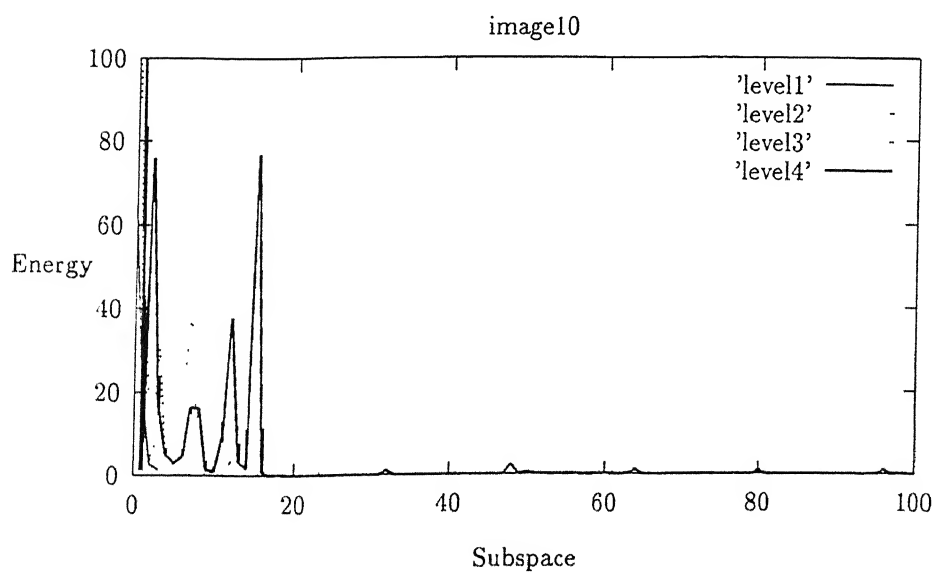


Figure 3.21: Energy distribution Image10

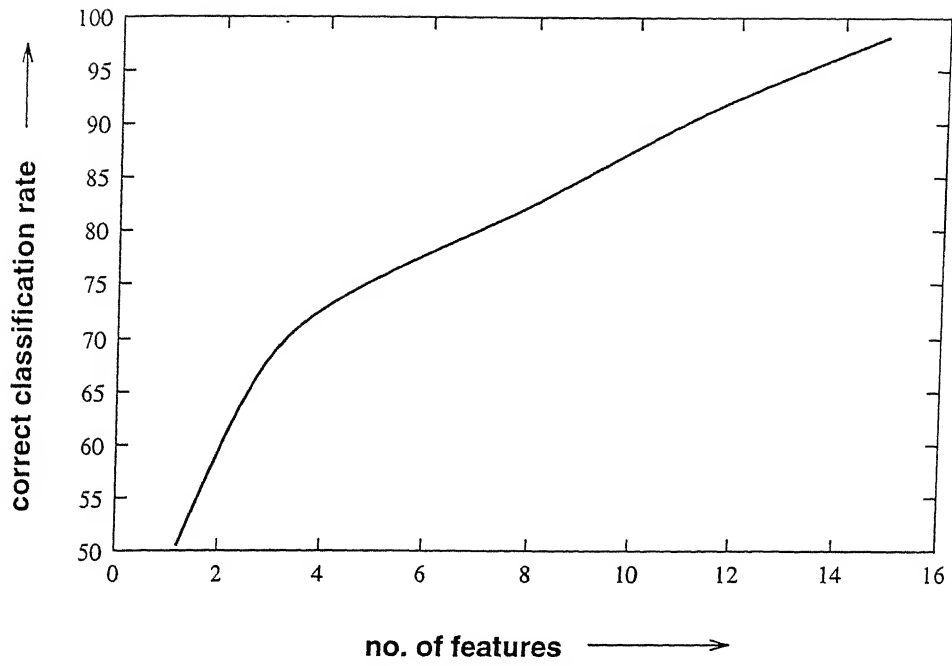


Figure 3.22: Classification rate Daubechies 4-tap Filter

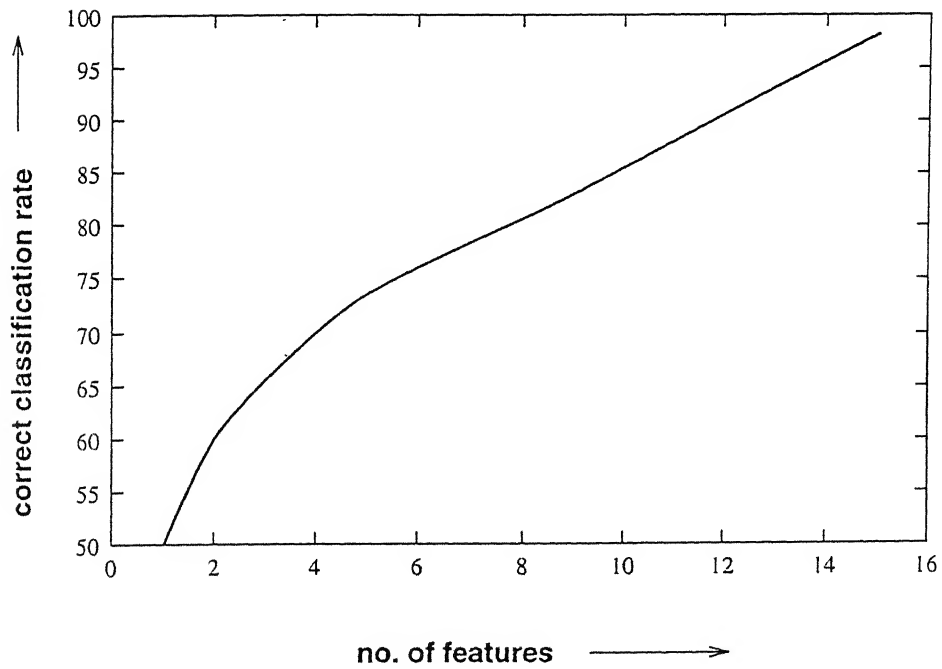


Figure 3.23: Classification rate Daubechies 20-tap Filter

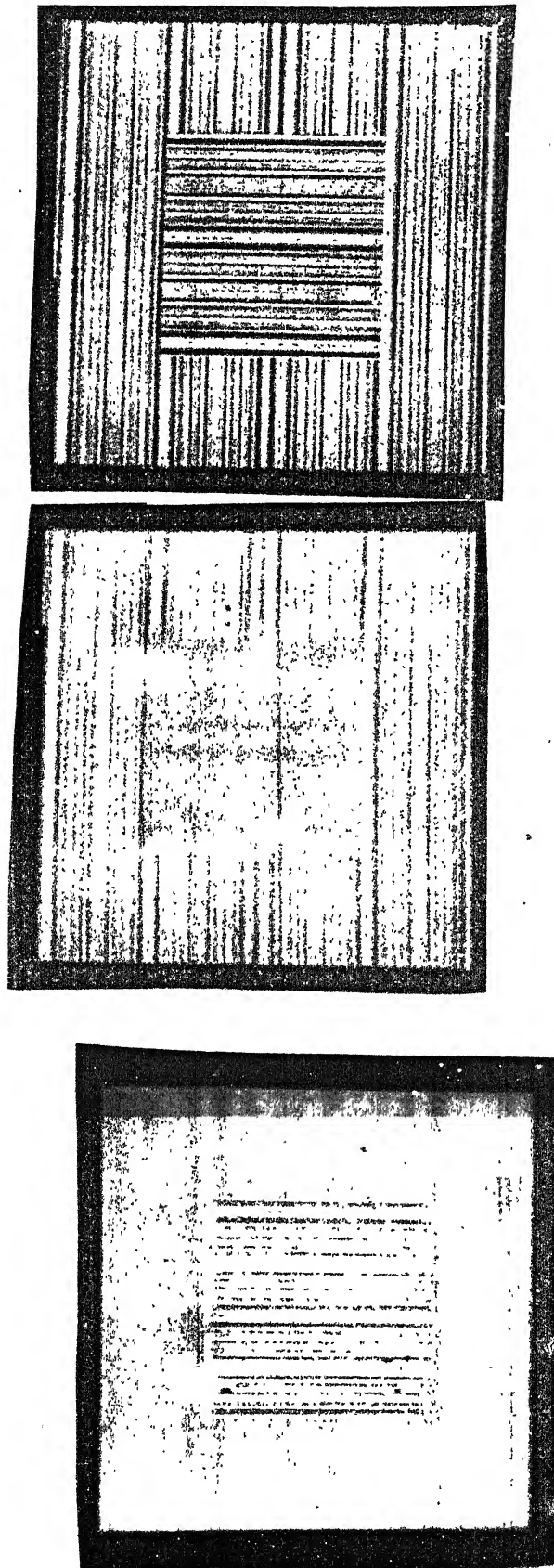


Figure 3.24: Region Identification Two Textured Images Set-1

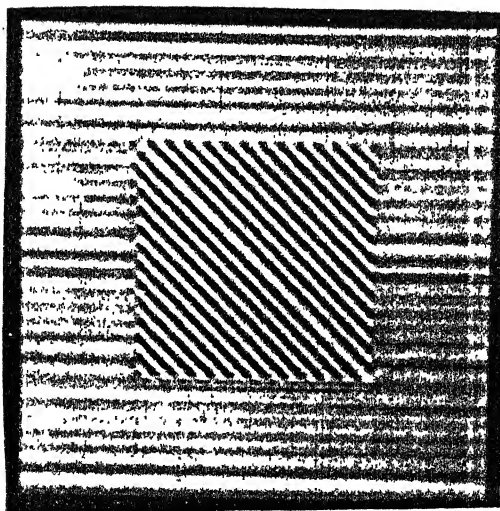
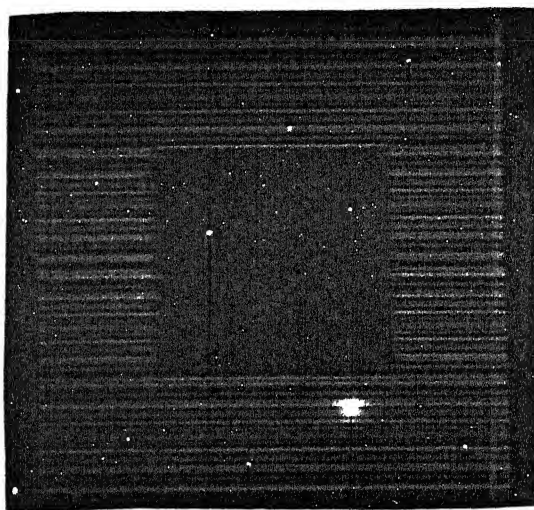
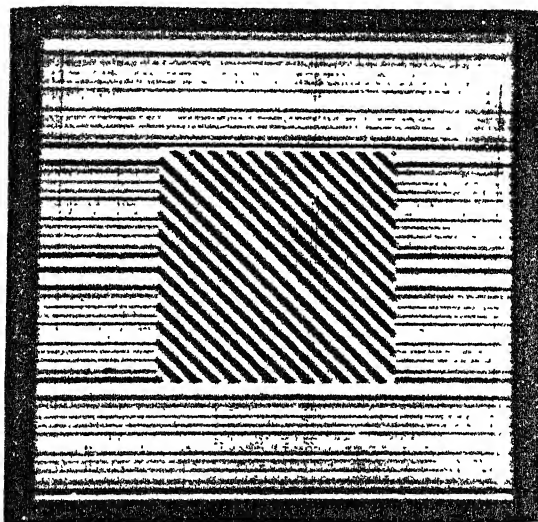


Figure 2.25. Region Identification Two Textured Images Set-2

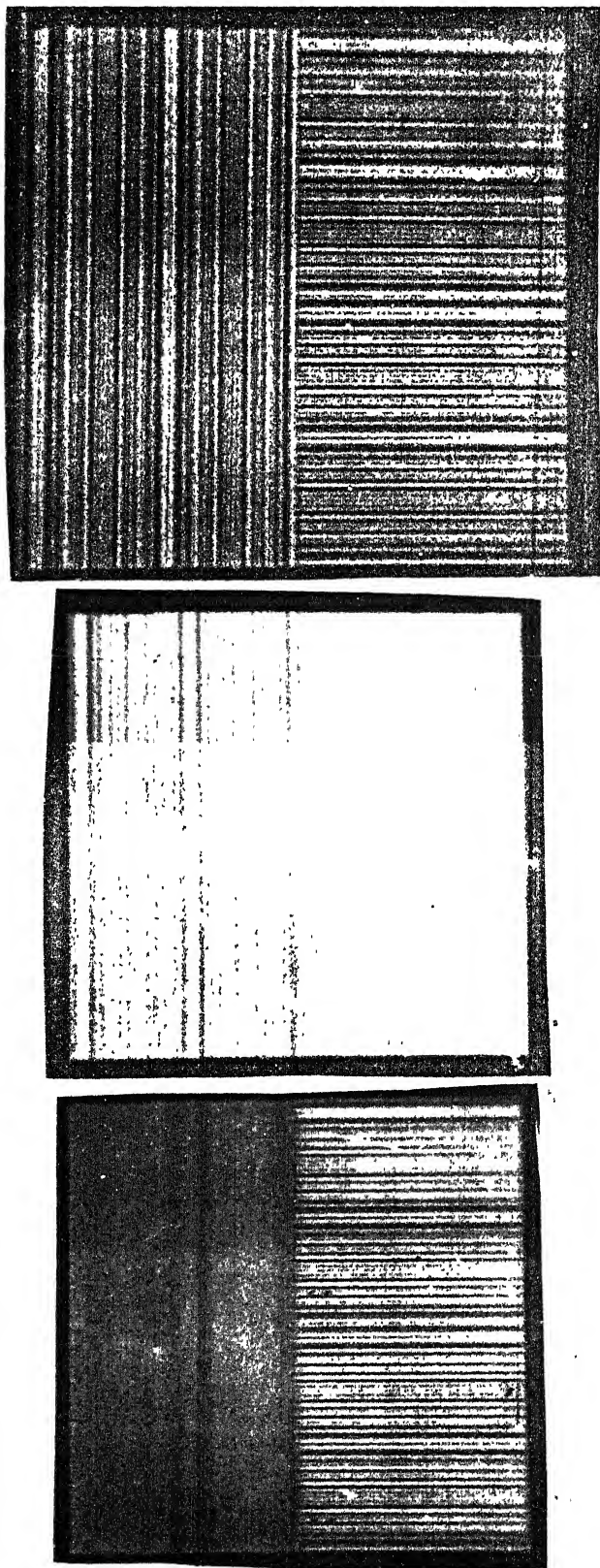


Figure 3.26: Region Identification Two Textured Images Set-3

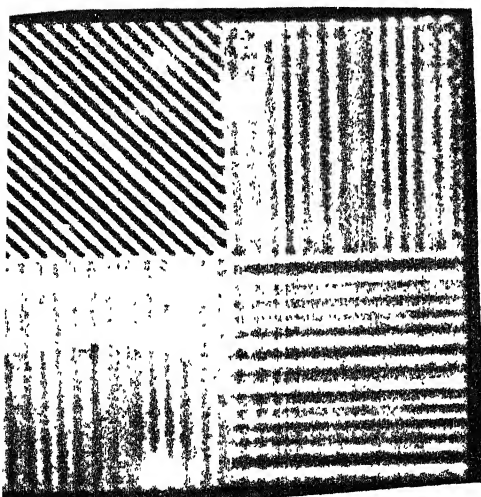
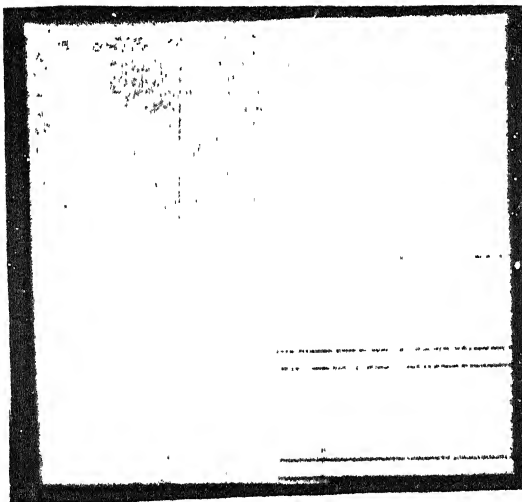
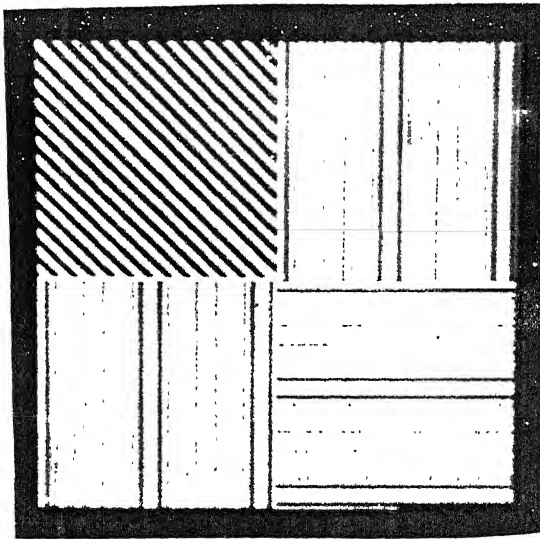


Figure 3.27: Region Identification Three Textured Images

Chapter 4

Conclusions

Directionality is an important characteristic of the texture. Though various statistical and structural techniques have been in use for a long time, multiresolution analysis, using wavelet decompositions, is proving to be very effective in texture analysis. In this work the use of wavelet packet transforms for texture classification of single textured images, region identification and classification for two and three textured images with distinct directional orientations was investigated. For classification, two algorithms were studied. One was the progressive classification algorithm and the other complete energy map classification. Due to the multi resolution capability of the wavelet packet transforms we can concentrate on the desired frequency locations and carry out the texture classification and identification. The texture locations were also changed to observe the performance for region identification. It was observed that the region identification was still done. Also since the wavelet packet transforms carry out scale space decomposition for both the low as well as high frequency components of the image they have better discriminability unlike the wavelet transforms which concentrate only on the low frequency portion of the signal.

4.1 Suggestions For Future Work

There is tremendous scope for future work in this field.

- ④ The texture classification using the entropy criterion can be investigated. Already the entropy criterion is used for coding by the best basis selection of wavelet packet subspaces.
- ④ The Two dimensional separable wavelet packet transforms give the discriminability in three different directions. By using the non -separable wavelet transforms and hexagonal wavelets a larger number of directions can be identified. This will given greater choice of texture discrimination.

Bibliography

- [1] J. Daubechies, "Orthonormal Basis Of Compactly Supported Wavelets" *Communications on Pure and Applied Math*, Vol 41, no 7, Oct 1988 , pp 909-990.
- [2] S. G. Mallat , "A Theory For Multiresolution Signal Decomposition" , *IEEE Trans on Pattern Analysis and Machine Intelligence* , Vol II , No 7 , July 1989 , pp 674-693 .
- [3] Olivier Rioul and Martin Vetterli , " Wavelets and Signal Processing " , *IEEE SP Magazine* " , Oct 1991 , pp 14-38.
- [4] " Special Issue On Wavelet Transforms " , *IEEE Trans. on Information Technology* , Vol 38 , No 2 , Mar 1992 .
- [5] Tianhorng Chang and C.-C.Jay Kuo , "Texture Analysis and Classification with Tree-Structured Wavelet Transform" , *IEEE Trans on Image Processing* , Vol " , No 4, Oct 1993 .
- [6] R. M. Haralick , " Statistical and Structural Approaches to Texture " , *Proc IEEE* , Vol 67, May 1979 , pp 786-804 .
- [7] R. M. Haralick, K. Shanmugam and I. Dinstein , " Textural Features for Image Classification" , *IEEE Trans System, Man., Cybernetics* , Vol SMC-6 , 1976, pp 269-285.
- [8] L. S. Davis, M Clearman and J. K. Aggarwal , "An empirical evaluation of Generalized Cooccurrence Matrices " , *IEEE Trans Pattern and Machine Intelligence* , PAMI-3, No 2, 1981 ,pp 214-221.

- [9] R. W. Connors and C. A. Harlow , “ A Theoretical Comparison of Texture Algorithms ” , *IEEE Trans Pattern and Machine Intelligence* , PAMI 2 , No 3 , 1980 , pp 204-222 .
- [10] M. M. Galloway , “ Texture Analysis Using Gray Level Run Lengths ” , *Computer Graphics Image Processing* , No 4 , 1975 , pp 172-179 .
- [11] A. K. Jain , “ Fundamentals of Digital Image Processing ” , Englewood Cliffs , NJ: Prentice Hall , 1979 .
- [12] L. Van Gool, P. Dewaele and A. Oosterlinck , “ Texture Analysis Anno 1983 ” , *Computer Vision Graphics and Image Processing* , No 29 , 1985 , pp 336-229 .
- [13] F. Tomita , Y. Shirai and S. Tsuji , “ Description of Textures by a Structural Analysis ” , *IEEE Trans Pattern Anal. and Machine Intelligence* , PAMI-4 , No 2 , 1982 , pp 183-191 .
- [14] S. W. Zucker , “ Towards a Model of Texture ” , *Computer Graphics Image Processing* , No 5 , 1976 , pp 190-202 .
- [15] P. de Souza , “ Texture Recognition via Autoregression ” , *Pattern Recognition* , 15 , No 6 , 1982 , 471-475 .
- [16] R. Chellappa , “ Two Dimensional Discrete Gaussian Markov Random Field Models for Image Processing ” , *Pattern Recognition* , Vol 2 , 1985 , pp 79-112 .
- [17] A. C. Bovik , “Analysis of Multichannel narrow-band filters for Image Texture Segmentation ” , *IEEE Trans Signal Processing* , Vol 39 , Sept 1991 , pp2025-2043 .
- [18] C. Bouman and B. Liu , “ Multiple Resolution Segmentation of Textured Images ” , *IEEE Trans Pattern Anal. and Machine Intelligence* , Vol 13 , Feb 1991 , pp 99-113 .
- [19] A. C. Bovik , M. Clark and W. S. Geisler , “ Multichannel Texture Analysis using Localized Spatial Filters ” , *IEEE Trans Pattern Anal. and Machine Intelligence* , Vol 12 , Jan 1990 .

- [20] R. R. Coifman and M. V. Wickerhauser , “ Entropy-based Algorithms for Best Basis Selection” , *IEEE Trans Information Theory* , Vol 38, March 1992 , pp 713-718 .

6 12 17 5

EE-1996-M-PRA-REG